



Universität der Künste Berlin

Universität der Künste Berlin
Studio für Intonationsforschung und mikrotonale Komposition
MA Komposition
Wintersemester 2020–21

Masterarbeit
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Surprising Connections in Extended Just Intonation

Investigations and reflections on tonal change

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Berlin, den 24.1.2021

Surprising Connections in Extended Just Intonation

Investigations and reflections on tonal change

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Bachelor of Music in Composition and Theory, University of Victoria, 2017

A dissertation submitted in partial fulfilment
of the requirements for the degree of

MASTER OF MUSIC

in the Department of Composition

January 24, 2021

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Abstract

This essay documents some initial speculations regarding how harmonies (might) evolve in extended just intonation, connecting back to various practices from two perspectives that have been influential to my work. The first perspective, which is the primary investigation, concerns itself with an intervallic conception of just intonation, centring around Harry Partch's technique of *Otonalities* and *Utonalities* interacting through *Tonality Flux*: close contrapuntal proximities bridging microtonal chordal structures. An analysis of Partch's 1943 composition *Dark Brother*, one of his earliest compositions to use this technique extensively, is proposed, contextualised within his 43-tone "Monophonic" system and greater aesthetic interests. This is followed by further approaches to just intonation composition from the perspective of the extended harmonic series and spectral interaction in acoustic sounds. Recent works and practices from composers La Monte Young, Éliane Radigue, Ellen Fullman, and Catherine Lamb are considered, with a focus on the shifting modalities and neighbouring partials in Lamb's string quartet *divisio spiralis* (2019). Finally, I connect this discussion to my current compositional interests, which have been exploring a method of microtonal modulation through arbitrarily near enharmonic connections in Harmonic Space called "enharmonic proximities".

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Acknowledgements

Throughout the writing of this essay, I have received a great deal of support, guidance, and assistance from my supervisor, Marc Sabat, whose expertise in the areas of music composition and just intonation research/practice have been invaluable to my creative and theoretical developments. I look forward to our many future collaborations to come.

I would also like to thank Professor Dr Ariane Jeßulat for her thoughtful advice on how to approach a musicological/theoretical project as a creative artist.

Finally, I would like to thank Kimberley Farris-Manning, whom I bombarded at various times throughout the development and writing of this essay with ideas and passages for feedback and discussion.

Foreword

Experiencing and coping with *change* seem to be intrinsic aspects of the human condition. Few processes—whether physical, societal, cognitive, etc.—are fundamentally *unchanging*, and we seem to actually seek out and take pleasure in multiple forms of change: gradual change (e.g. the satisfaction of saving small amounts of money over many years); sudden change (e.g. rays of sunlight breaking the cloud cover on a bleak day); expected change (e.g. a climactic shift in a favourite piece of music); unexpected change (e.g. a shocking plot twist in a book or film). Specifically, change *activates* our modalities of perception, and thus plays a crucial role in our processing of structure in sensory experiences. This especially seems the case for our engagement with art, in particular music, a complex experience *in time* and *of time*. Forms of temporality begin to emerge as partitions are constructed by perception, resulting from the interplay between the absolute periodicity of the “arrow of time” and the artificially and imperfectly constructed events/procedures mediated by pure human creativity (“musical time”).

For Gérard Grisey, the first distinct change in a piece of music was the most immediate challenge for a composer, gauging the listener’s subsequent engagement with the composition’s broader continuity.¹ “Everybody can have an idea...The problem is to have a second one. And the major problem is to know where and when to bring in this second idea.”² Though the frankness of this sentiment might make it seem like an oversimplification, it is really at the heart of our eternal dance with change. *When* change occurs can be a far more deciding factor in our experience of it, its effect on us, over *what* the change is.

¹ Justyna Humięcka-Jakubowska, “The Spectralism of Gerard Grisey: From the Nature of the Sound to the Nature of Listening,” *Interdisciplinary Studies in Musicology*, no. 8 (2009): 240, <http://cejsh.icm.edu.pl/cejsh/element/bwmetal.element.ojs-issn-1734-2406-year-2009-issue-8-article-14916>.

² David Bündler, “Interviews: Gerard Grisey,” 1996, <https://www.angelfire.com/music2/davidbundler/grisey.html>.

When I first knowingly experience microtonality, specifically *just intonation*, what initially drew me with such strength were not the discrete sounds per se—a pure triad, the harmonic series “chord”, periodic beating—but rather the strange, magical sensation caused by one sound *changing* into another sound, an aggregate of tones *becoming* another, and how, at just the right moment within some musical continuity, such a tonal shift can have as profound and lasting an effect as (some might say) “an entire symphony”. It was immediately clear that the pervasive uniformity of temperament (whether 12-tone, 24-tone, any in common use) was simply incapable of quite recreating such wonderful transformations.

Afterwards, I came to realise that this feeling was stimulated by the far more nuanced interaction of the rich assortment of *differently sized* intervals characteristic of *extended just intonation*. Pioneered in the early twentieth century by Elsie Hamilton and Kathleen Schlesinger, extended just intonation, as the name suggests, is an extension of the “pure” intonation most listeners associate with the crystalline tuning of Renaissance polyphony. Rather than being limited to frequency relationships with no prime factors exceeding 5, the principle of extended just intonation pushes into ever higher and, in a sense, more complex prime territory: embracing 7, 11, 13, and so on. For centuries, these higher primes were a source of contention between composers and theoreticians, some of whom deemed them *unsuitable* for musical purposes, others a source of great harmonic subtlety,³ and yet the grand network of tonal nuances they suggest is exactly the framework underlying the phenomena I and an ever-growing number of other musicians find so appealing about extended just intonation—and utterly unique to it. Surely that is reason enough to justify their musical exploration.

I have since dedicated much of my artistic energies to researching the seemingly infinite manners in which tonal change might take place in a modulating extended just intonation framework, extracting those options that appeal the most to me from a purely sonic perspective. And more importantly, I find myself in an ongoing process of learning to compose with such relationships in musically and personally meaningful ways: “where and when” to bring them in. On one hand, the inner workings of such a complex, multidimensional topic have a necessarily corresponding degree of technical intricacy, however, the sensory reward provided by the musical product far outshines any suspected Pythagoreanism and provides insight into the intrinsic poetry in the numbers. In particular, two approaches taken by other composers over the last century—namely *Tonality Flux* (Harry Partch) and the activation of interactions between harmonic spectra high in the series (La Monte Young, Éliane Radigue,

³ *Contra*: Gioseffo Zarlino, Jean-Philippe Rameau, Hermann von Helmholtz, Paul Hindemith, Alain Daniélou, etc.—*Pro*: Gottfried Wilhelm Leibniz, Leonhard Euler, Giuseppe Tartini, Carl Stumpf, Harry Partch, etc.

Ellen Fullman, Catherine Lamb)—have been influential on a practice I have developed for myself, which I describe in detail in this essay.

For me, composing in extended just intonation is a seeking out of the unique sonorities and unparalleled expressive interplay inherent to *finely differentiated* and *tuneable* melodies and harmonies. It is an affirmation and an appreciation of the intricate *unevenness* that such a system demands, and a desire to create music that nears ever closer to the way humans actually perceive the interaction of tones.

Chapter 1: Some Preliminary Considerations

Broadly speaking, *microtonality* might refer to differentiation in the perception of fine shadings of intonation, whether intentional or unintentional. In this sense, it seems to me that the human voice is the instigator *par excellence* of microtonality, being neither constrained to any particular tonal centre nor any fixed scale or limited gamut of tones. It is capable of freely gliding between tones (frequencies, rates of vibration) unlike almost any other instrument—with the potential to articulate any point along the tonal continuum. Only fretless stringed instruments, perhaps, approach this kind of mechanical flexibility. However, no instrument “external” to the human body is quite able to provide the same sense of embodiment that one’s own voice provides. As humans, our voices are integral to our socio-physiology, being an intrinsic aspect of our *auditory* communication.

1.1 Microtonality in communication and its influence on music

Perhaps most pertinent to this assertion is the simple fact that human *speech* is inherently microtonal. Aristoxenus, writing ca. 300 BC, describes speaking as a smooth variation in pitch. Speech “seems to the senses to traverse a certain space in such a manner that it does not become stationary at any point, not even at the extremities of its progress...but passes on into silence with unbroken continuity.”⁴ While the voice does (for the most part) glide between points of tonal articulation, it does not do so at a constant rate. A frequency analysis of my own speaking (fig. 1) reveals not only its gliding and “step-wise” microtonality but also variations in the rate of change as well as discontinuities caused by articulations and gaps between words. As Aristoxenus describes, speech never becomes *stationary* as such, but rather articulates regions of *lingering* and *transition*.

⁴ Aristoxenus, *The Harmonics*, Henry S. Macran (London: Oxford: At the Clarendon Press, 1902), 170–71.

How microtonal features of speech are parsed is evidently a complex question and a truly comprehensive discussion of the relevant mechanisms is beyond the scope of this essay. Nevertheless, certain aspects seem to me to be clearly at work. In communication, microtonality is bound together with other auditory phenomena, such as emphasis and unvoiced articulations, and interpreted on multiple levels by the brain. Various types of meaning are extracted, ranging from semantic to emotional. The correct order of phonemes successfully conveys the intended word, while dynamics, register, melodic cadence, as well as timbral/spectral balance shade and potentially alter the semantic content. Subtle variations in pitch play a crucial role throughout this process. The exact way in which these subtleties work is largely language and context-dependent, relying on lexical “libraries” and acquired “templates” or “prototypes.”⁵⁶ Consider how, in English, the inflexion at the end of a sentence may turn a statement into a question or how the fine line between a genuine and a sarcastic remark is heavily dependent on intonation.

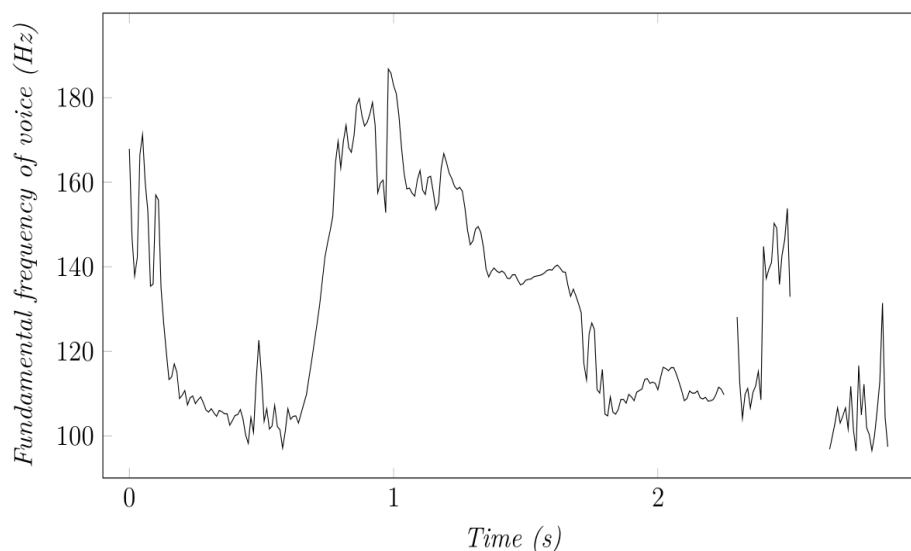


Figure 1: Frequency analysis (with SPEAR) of my voice as I speak the sentence “I neither leave nor arrive on the midnight train”; illustration of how the fundamental frequency (i.e. perceived pitch) drifts as a function of time.

The process of generating (creating) and consuming (understanding) both spoken language and *music* are closely related: the sound is produced—the sound is perceived—the sound is parsed. It appears that the difference at least partially lies on the semantic level, contingent on

⁵ Patricia Kuhl, “Human Adults and Human Infants Show a ‘Perceptual Magnet Effect’ for the Prototypes of Speech Categories. Monkeys Do Not.” *Perception & Psychophysics* 50 (1991): 93–107, <https://doi.org/10.3758/BF03212211>.

⁶ Robert J. Sternberg and Karin Sternberg, “Language Comprehension,” in *Cognitive Psychology*, 6th ed. (Belmont, Calif: Cengage Learning, 2011), 368–85.

whether or not most of the sounds fit a familiar language as phonemic units. When they do not, we seem to more readily allow ourselves to listen to sounds *musically*, i.e. without the need to search for semantic information even when the sound is intended as language-communication within some unfamiliar criteria. The concepts of *birdsong* and *whale song*, in addition to the vocalisations of many other animals, clearly fall in this category. Often, this is also the case when we hear foreign (or invented) languages, even despite our awareness of a *potentially knowable* lexicon and grammar (cf. Kurt Schwitters's *Ursonata* and in particular Christopher Butterfield's notably microtonal recording⁷). This cognitive interplay seems to underlie many stereotypes, such as the melodic lilt that English-speakers associate with Scandinavian languages. In essence, semantically unparsed human vocal utterance is perceived on a continuum mediated by one's own musical templates. Conceivably, musical listening may even be allowed to *override* semantic involvement with language itself, like how we quickly overlook linguistic content when listening to John Cage's *Roratorio*, focusing instead on an "orchestration" of human-made utterances interacting with all sorts of natural and mechanical sounds and noises.

Since microtonality is characteristic of auditory communication through speech as well as essential to its comprehension, it must have the potential to become a salient and integral aspect of *new* forms of musical expression.

1.2 The question of tuneability: just intonation and temperament

Aristoxenus writes quite plainly in his *Elementa Harmonica*, "we hold that the voice follows a natural law in its motion, and does not place the intervals at random."⁸ Its method must appeal to *both* hearing and intellect, judging "the magnitudes of the intervals" and "the functions of the notes" respectively.⁹ Generally, Aristoxenus is sceptical of the older Pythagorean resolution that the ultimate parameter of "harmony" is scientific reason alone—the whole number relations of string lengths (i.e. frequencies), specifically those built from combining powers of 2s and 3s—stressing a more "musical" point of view. In Henry S. MacRan's notes to his English translation of the ancient text, the editor summarises this difference quite concisely.

The contrast between the Pythagorean and Aristoxenian views of musical science comes out strongly in the definitions of a tone. For the Pythagoreans a tone is the

⁷ Christopher Butterfield, "Music for Klein and Beuys / Ursonate / Pillar of Snails" (Canada: Artifact Music, 1993), tracks 2–5.

⁸ Aristoxenus, *The Harmonics*, 188.

⁹ Aristoxenus, *The Harmonics*, 189.

difference between two sounds whose rates of vibration stand in the relation 8:9; for the school of Aristoxenus, the difference between a Fourth and a Fifth. The latter explain the phenomena of music by reducing these to more immediately known *musical* phenomena, the former by reducing them to their mathematical antecedents.¹⁰

Typically, this distinction is understood as a complete rejection of ascribing importance to rational frequency ratios, but I would assume a more subtle interpretation of Aristoxenus's position. Without much exposition, he attributes the "concordance" of the main structural intervals of his musical system (i.e. the fourth, fifth, octave) to a sensation simply known and shared by all musicians—on the other hand, the ear is much less "assured" of the "discords" (e.g. tone, "ditone" [major third]), usually requiring some kind of constructive procedure, i.e. composition, by means of the concords to produce them.¹¹ Elegantly translated by John Hawkins, Aristoxenus proposes, "just as it is not necessary for him who writes an Iambic to attend to the arithmetical proportions of the feet of which it is composed, so it is not necessary for him who writes a Phrygian song to attend to the ratios of the sounds proper thereto."¹² In essence, the particular sensations associated with simple proportions come innate, pre-programmed in perception and a musician need not concern themselves with the numerical details. The smallest intervals, in particular—the quarter-tones and third-tones characteristic of the Greek genera—divide the tone into approximately equal parts whose exact intonation is principally influenced by musical expression and taste: "to employ the third part of a tone is a very different thing from dividing a tone into three parts and singing all three."¹³

Ptolemy expands greatly on this framework in the second century, taking a kind of middle ground in his *Harmonics*. A true understanding of harmony must be informed by both scientific and musical intellect (and even intuition): perception must confirm reason and agree with it, but it must not be allowed to become complacent, its judgements left unquestioned.

The criteria of *harmonia* [harmony] are hearing and reason, not however in the same way...Hearing is concerned with the matter and the modification, reason with the form and the cause, since it is in general characteristic of the senses to discover what is approximate and to adopt from elsewhere what is accurate, and of reason to adopt from elsewhere what is approximate, and to discover what is accurate. ...Since matter is determined and bounded only by form, and modifications only by the causes of movements, and since of these the former...belong to sense perception, the latter to reason, it follows naturally that the apprehensions of the senses are determined and bounded by those of reason, first submitting to them the

¹⁰ Aristoxenus, *The Harmonics*, 245.

¹¹ Aristoxenus, *The Harmonics*, 206.

¹² John Hawkins, *A General History of the Science and Practice of Music* (Novello, Ewer & Company, 1875), 67.

¹³ Aristoxenus, *The Harmonics*, 199.

distinctions that they have grasped in rough outline...and being guided by them towards distinctions that are accurate and accepted. This is because it is a feature of reason that it is simple and unmixed, and is therefore autonomous and ordered, and is always the same in relation to the same things, while perception is always involved with multifariously mixed and changeable matter, so that because of the instability of this matter, neither the perception of all people, nor even that of the very same people, remains the same when directed repeatedly to objects in the same condition; but it needs, as it were as a crutch, the additional teaching of reason.¹⁴

Ptolemy likens this to a free-hand drawing of a circle. Initially, it may appear to be accurate, but upon comparison with one “formed by means of reason”—i.e. with a compass—its inadequacy is made clear to the senses. Perception, possessing (to a lesser or greater degree) an intuition for relations of tones, may be made more discerning—particularly when comparing fine differences—by studying its association with numerical proportions, themselves reverified in practice by that same perceptual mechanism, and so on. The implication is a musical culture surrounding a symbiotic dependency linking concrete listening to abstract reasoning.

In his *Kitab al-Musiqā al-kabir* (Great Book of Music), tenth-century philosopher and music scholar Abu Nasr al-Farabi details the abundance of microtonality in Arabic music theory, giving multiple arguments and examples based on its precedence in contemporary practice. Noting that “in actuality, there exist notes [relationships] that are natural to the ear [i.e. perceptible], which no resonant body—whether string, human voice, or any other instrument—is able to produce,”¹⁵ he insists that the gamut of tones available on an instrument ought to be those that, within its own restrictions, may best imitate the tones produced by the human voice. Al-Farabi describes with exactitude how an oud player should stop the strings at multiple *specific divisions* to simulate the flexibility and nuance of the voice. His specifications produce an *unequal* scale with 22 degrees,¹⁶ shown in fig. 2, a reconstruction of al-Farabi’s original diagram, and fig. 3, the diagram converted into staff notation using ♮G as the lowest string. Similarly, the thirteenth-century musicologist Sarngadeva explains that, in the classical traditions of India, the octave is also divided into 22 tones. He writes, “Nada [musical sound] is differentiated into twentytwo [sic] grades which, because of their audibility, are known as *srutis*.”¹⁷ Both writers stress the importance of *perception* as a parameter when defining tonal gamuts for music. Only those relations of tones that are

¹⁴ Andrew Barker, *Greek Musical Writings, Volume II: Harmonic and Acoustic Theory* (Cambridge: Cambridge University Press, 1997), 276–77.

¹⁵ Abu Nasr al-Farabi, *Grand Traité de La Musique (Livres I et II)*, trans. Rudolf d’Erlanger, La Musique Arabe 1 (Paris: Librairie orientaliste Paul Geuthner, 1930), 42.

¹⁶ Abu Nasr al-Farabi and Mahmoud Ahmed el-Hefny, *Kitab Al-Musiqā Al-Kabir*, ed. Ghattas Abd-el-Malek Khashab, 1st ed. (Tihiran: The Arab Writer, Cairo, 1975), 129.

¹⁷ Sarngadeva, *Sangita-Ratnakara*, trans. R. K. Shringy (Delhi: Motilal Banarsidass, 1978), 115.

“natural to the ear” and “discerned in the throat as well as in the cerebrum”¹⁸ are considered adequate for musical use.

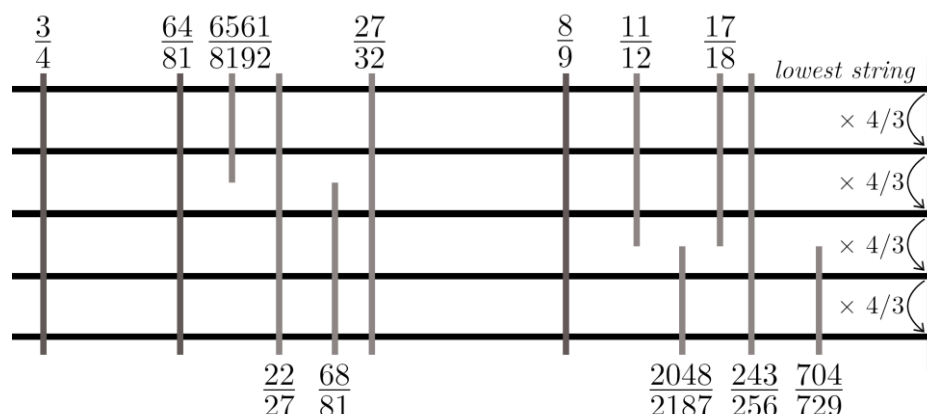


Figure 2: Al-Farabi's diagram showing the divisions where he suggests stopping the five strings of the oud. Note how the strings—tuned in fourths—require different patterns of divisions to ensure that both octaves of the instrument have the same gamut of tones, a requirement that al-Farabi takes for granted: “It is evident that the notes of the second octave must replicate those of the first.”¹⁹

Note that perceptibility correlates with *tuneability*—if a relationship between tones may be unambiguously perceived, then it must be reproducible under adequate conditions and with adequate physical skill, verified by the same faculties that were able to perceive it in the first place. Particularly for basic consonances, the notion of *in-tune-ness* is closely linked to *rational* relationships of frequencies: those that are (very nearly) in small whole number proportions, whose periodic resonance is unambiguously parsed by the brain.

¹⁸ Sarngadeva, *Sangita-Ratnakara*, 116.

¹⁹ Al-Farabi, *Grand Traité*, 45.

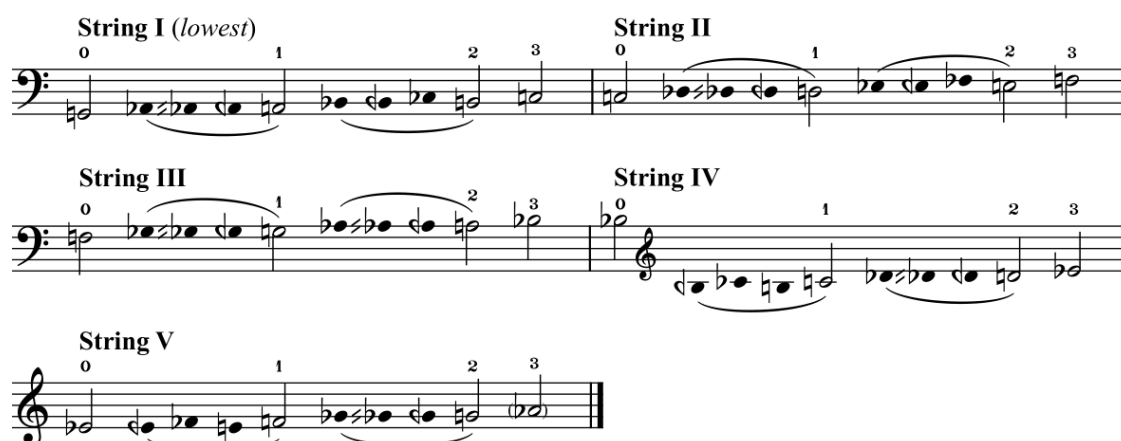


Figure 3: The resulting scale of al-Farabi's finger positions, notated in Helmholtz-Ellis JI Pitch Notation (which will be discussed in more detail in chapter 2).

Hermann von Helmholtz ranks acoustic consonance—or, possibly, *concordance* to distinguish it from *musical consonance*—based on the elimination and/or stabilisation of *beating* (interference, friction) between coinciding spectral partials and combinational tones of intervals wider than the critical band (between a tone and a minor third in the upper-mid register).^{20,21} For intervals and chords of rich tones, such as those produced by the voice and most acoustic instruments, this form of interaction between the tones' spectra reinforces our sense of the overall waveform's periodicity. Intervals like the octave, whose frequencies are in the ratio 2:1, and the perfect twelfth 3:1 exhibit strong *smoothness* because all of the partials of the upper tone coincide exactly (as unisons) with partials of the lower tone. This has been illustrated in fig. 4 (a) and (b) respectively. As intervals become more complex (that is, roughly, as the magnitude of the numbers grows), a smaller percentage of each tone's spectrum is exactly coincident with that of the other tone(s), resulting in the possibility of many more accentuated interactions between partials inside the critical band. For instance, two tones in the relationship 7:6—itself already on the edge of, or slightly within, the critical band—exhibit less smoothness than the relationships previously mentioned. Only every sixth partial of the higher tone's spectrum coincides in unison with a partial of the lower tone's

²⁰ Hermann von Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, trans. Alexander J. Ellis, Third (New York: Longmans, Green, & Co., 1895), 181.

²¹ The resonant regions on the ear's basilar membrane of tones comprising intervals within the critical band begin to overlap as the frequency distance between the two tones approaches a unison. Such regions interfere with one another, causing beats of a single perceived average frequency, or—as beating moves beyond ca. 15 Hz—a sensation of roughness between two very similar frequencies. This is characteristic of all intervals smaller than the critical band. Below 500 Hz, the size of the band itself is a roughly constant frequency difference of ca. 100 Hz, thus becoming progressively wider in terms of the perceived interval in the lower register. For more information, refer to Juan G. Roederer's *The Physics and Psychophysics of Music*, sec. 2.4.

spectrum, illustrated in fig. 1.4 (c). Furthermore, even before the very first coincidence (the lowest common partial shared between the two tones), there are many interactions between partials within the critical band, contributing to an increased sensation of *roughness*, as Helmholtz would describe it.

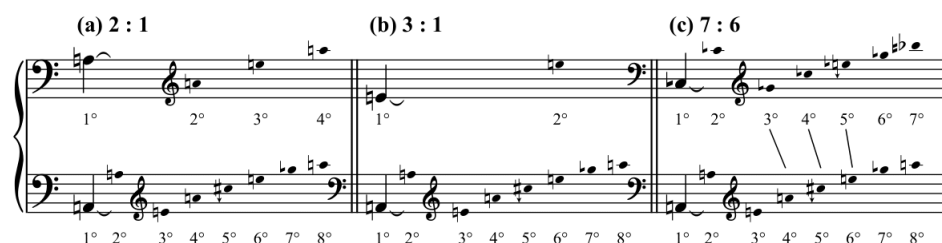


Figure 4: Comparison of harmonic partials between two tones forming the intervals 2:1 (a), 3:1 (b), and 7:6 (c). With fundamentals in the indicated register—provided for mere convenience of reading—7:6 falls undoubtedly within the critical band, and would require transposition to a higher register to begin to exhibit its maximum smoothness. Further critical band interactions between partials leading up to the lowest common partial (for fundamentals in any register) are connected with a line.

Carl Stumpf generalises Helmholtz by relating perceived concordance and discordance with the degree to which a complex of tones *fuses*, representing the spectrum of a single tone (which may or may not be sounding).²² This is twofold: concordance is maximised as either the complexity of the numerical relationships (*not* the size of the interval) approaches a unison ($3:2 \rightarrow 3:1 \rightarrow 2:1 \rightarrow 1:1$), or, for larger complexes, as more and more of the harmonic series of a single tone (fundamental) is represented. For a much more comprehensive overview of theories of “consonance and dissonance”, I recommend referring to James Tenney’s *A History of Consonance and Dissonance*.²³ In general, the vast continuum of whole number ratios and proportions of frequencies, which are analogous to the tuning of aggregates of partials of a harmonic series and whose *relative dissonance* (or consonance) lies on a finely grained spectrum, is typically given the name *just intonation*.

When comparing the various sizes of ratios, it may not be immediately obvious which ratios are larger than others, nor to what extent. Frequency space is perceived logarithmically, which is why a ratio of frequencies are heard as a constant interval in all registers. The amount by which two ratios differ is only found by dividing them; to combine ratios, they must be multiplied together. Mathematician (and translator of Helmholtz) Alexander J. Ellis proposed the division of each octave into 1200 very small, equal units of measure called *cents*.²⁴

²² Albrecht Schneider, “‘Verschmelzung’, Tonal Fusion, and Consonance: Carl Stumpf Revisited,” in *Music, Gestalt, and Computing*, ed. Marc Leman, Lecture Notes in Computer Science (Berlin, Heidelberg: Springer, 1997), 126–27.

²³ James Tenney, *A History of Consonance and Dissonance*, 1st ed. (UK: Routledge, 1988).

²⁴ For a ratio $a:b$, the cent value (c) is defined to be $c = 1200 \times \log_2(a/b)$.

Incremental changes would be imperceptible to the human ear,²⁵ but cents might serve as a high-resolution linear measure of ratios, like an “auditory ruler”. The majority of electronic tuners and tuning apps include a measure of an input frequency’s cent *deviation* from the standard equal-tempered semitones (i.e. the amount by which a tone differs from the nearest equal-tempered chromatic pitch). Cents provide an extremely practical counterpart to the more abstract ratio form of an interval because cent values may be simply added and subtracted as intervals rise and fall. Ellis’s measure may be used to compare some of the intervals already discussed in this chapter (tbl. 1). Most of these intervals are very near to equal temperament, however, 7:6 is revealed to be a very narrow minor third, 33 cents lower than the equal-tempered minor third (300 cents). From this table, it is also easy to see that the whole tone, which Aristoxenus explains is the difference between the perfect fifth and the perfect fourth, is $702 - 498 = 204$ cents, even without calculating the quotient to find its ratio (i.e. 9:8). Note, I will not go into any more detail about the basic mathematics pertaining to ratios and just intonation, for which there are already countless in-depth resources.²⁶ As such, this essay assumes a functional understanding of these concepts.

Ratio	Size in cents
3:1	1902 cents
2:1	1200 cents
3:2	702 cents
4:3	498 cents
7:6	267 cents

Table 1: Cent values (rounded to whole numbers) of some intervals discussed in this chapter.

Considering, briefly, some distant historical precedents in European musical practice relating to Helmholtz’s and Stumpf’s conceptualisations of concordance/discordance, Theinred of Dover (ca. 1100) and Walter Odington (ca. 1300) famously advocated for simple, concordant tuning of “imperfect” consonances (major/minor thirds and sixths). Odington observed that, in practice, even “if they [imperfect consonances] are discovered not to be consonant in number [i.e. proportions of very *large* numbers], the human voice nevertheless leads itself into

²⁵ Some musicians attest to being able to both perceive and physically produce a step of 1 cent (cf. bassoonist and composer Johnny Reinhard).

²⁶ For a more thorough discussion of these fundamental aspects of just intonation, refer to Marc Sabat and Thomas Nicholson, *Fundamental Principles of Just Intonation and Microtonal Composition* (Berlin: Plainsound Music Edition, 2018), <http://www.marcsabat.com/pdfs/JI.pdf>.

sweet mixture by its own subtlety.”²⁷ This is almost certainly a reaction to Boethius’s *De institutione musica*, which for hundreds of years served as the authoritative Latin-language source of Greek music-theoretical ideas. Though Boethius himself appears to echo Ptolemy’s stance that perception and logic must work together in the judging of harmonic relationships, he clearly sides asymmetrically with the Pythagorean camp, prioritising numerical “logic”. In *De institutione musica*, he writes, “although the sense of hearing recognizes consonances, reason weighs their value.”²⁸ Because of the Pythagorean position that the “melodic” [dissonant] intervals must be directly constructed from the consonances—the octave, the perfect fifth, and the perfect fourth—all intervals between the unison and the fourth, the fourth and the fifth, and the fifth and the octave are numerically complex, as they result from stacking fifths and fourths. Notably, those intervening intervals that we would categorise today as imperfect consonances sound particularly rough (i.e. discordant) in this “Pythagorean tuning”. Alternative options with much simpler ratios exist for these intervening intervals, especially involving the number 5 and its powers (e.g. the major third 5:4 over the Pythagorean ditone 81:64). Ptolemy points out that these are immediately favoured or at least more readily recognised by the ear over the Pythagorean variants. Nevertheless, Boethius rejects them, as they are “logically separated from the nature of consonance.”²⁹ Odington, who, himself under Boethius’s strong influence, even calls upon the numerical ratios of Pythagorean tuning when discussing the intervals,³⁰ provides a glimpse into *practice*, which had been branching away (maybe for some time) from a strictly Pythagorean framework. Of most immediate importance, his observation acknowledges an acceptance of ratios with 5 and its powers in European musical practice—perhaps, for the most part, through intuition.

Formally, however, attempting to unify proportions involving powers of 3 and 5 (so-called “5-limit”, because 5 is the largest prime number generating intervals) within a mutually consonant diatonic set gives rise to its own difficulties.³¹ Tuning the diatonic scale in this way requires at least 12 pitches to represent 7 notes if all consonances and imperfect consonances are meant to have concordant forms (fig. 5). For example, the slightly lowered sixth degree

²⁷ “*Et si in numeris non reperiantur consoni, voces tamen hominum sua subtilitate ipsos ducunt in mixturam suavem.*” Walter Odington, *Summa de Speculatione Musicae*, ed. Frederick F. Hammond, vol. 14, *Corpus Scriptorum de Musica* (Rome: American Institute of Musicology, 1970), 70–71.

²⁸ Anicius Manlius Severinus Boethius, *De Institutione Musica*, ed. Claude Palisca, trans. Calvin Bower, *Music Theory Translation Series* (New Haven: Yale University Press, 1989), 47.

²⁹ Boethius, *De Institutione Musica*, 48.

³⁰ He does this despite clearly implying the consonant tunings involving the number 5. One potential way of making sense of this is to simply consider the Pythagorean ratios as *labels* for interval classes.

³¹ No power of 3 is equivalent to a power of 5, so any tone system of proportions that mix both cannot be *closed*, i.e. contain a finite number of notes.

(5:3) will be discordant with the usual second degree (9:8), forming a narrow false fifth. To correct this, either the sixth must be microtonally raised or the second must be microtonally lowered.



Figure 5: The 5-limit diatonic scale on $\sharp D$, with the seven-note “base form” indicated with stems. Microtonal variants deviating by a syntonic comma and some of the harmonic relations to the base form that prescribe their inclusion are highlighted.

Over the centuries, various *temperaments*, such as the meantone family first formalised by Gioseffo Zarlino,³² were developed and played as a way of dealing with the growing tonal complexity. These efforts attempted to allow for the intersection of apparently incompatible worlds for the purposes of European musical practices. The “quarter-comma” variant of meantone temperament paints a clear picture, hinging on two principles: (1) that the ancient Pythagorean ditone (i.e. major third) may be constructed by stacking four perfect fifths (or, more directly, by stacking upward two perfect fifths and downward two perfect fourths); and (2) that the developing musical aesthetic was becoming increasingly more interested in thirds and sixths, thereby gravitating toward the most concordant tuning of these imperfect consonances, particularly for the sake of triads. In slightly falsifying—*tempering*—the perfect fifth by narrowing it a quarter of the difference of the Pythagorean ditone 81:64 and the concordant major third 5:4 (i.e. a quarter of the “syntonic comma”), a stack of four of such tempered fifths will produce a major third with the ratio 5:4 (fig. 6). Consequently, the major third is divided into two exactly equal parts, however, these parts are irrational due to tempering, each having a ratio of $(5:4)^{1/2} : 1$.³³

³² Specifically, Zarlino describes “2/7-comma meantone” temperament. Gioseffo Zarlino, *Le Istitutioni Harmoniche* (Venice, 1558), 126.

³³ By doubling each term of 5:4, we see that it may easily be divided into two rational but *unequal* parts, 10:9:8, as was the case in the base form of the 5-limit diatonic scale (fig. 5). This produces two sizes of whole tone, 9:8 and the slightly narrower 10:9. Quarter-comma meantone temperament “splits the difference” of these two sizes of whole tone, making use of a single, average (i.e. *mean*) tone equal to $(5:4)^{1/2}$.



Figure 6: A full 12-note chromatic gamut in quarter-comma meantone, branching outward from the generating tone $\flat D$. Each fifth is narrowed (or each fourth is widened) by a quarter of a syntonic comma, such that every fourth fifth is narrowed by a full syntonic comma to produce a concordant major third. Horizontal bars on the accidentals indicate tempered relationships from the pure perfect fifths with the portion of the syntonic comma tempered out indicated above. A full alteration by a syntonic comma is indicated by an arrow. All major thirds and minor sixths are exactly just, while all minor thirds and major sixths, like the perfect fifths and fourths, are nearly just (i.e. a quarter-comma too narrow and too wide respectively).

Indeed, for much diatonic music sung, for example, by well trained, unaccompanied voices, temperament is probably unnecessary and, to a large extent, unsubstantiated in practice. Even at the risk of modest amounts of pitch drift arising from the compounding of microtonal adjustments, groups of trained voices seek out concordance and have little practical concern for an increasing number of pitches. Rather, temperament's *modus operandi* is to establish common ground between the increasing interest in modulation and chromaticism as well as purely practical constraints, both mechanical and, as many musicians would doubtlessly advocate, mental. For one, the developing reliance on keyboard instruments, which needed to be able to accompany voices and other instruments with a relatively small, fixed number of notes.³⁴ While some highly specialised keyboard instruments were proposed, like Nicola Vicentino's two-manual *archicembalo* with numerous extra keys tuned in an extended quarter-comma meantone tuning,³⁵ the sheer tradition, if anything else, of the standard "chromatic" layout with only one tuning of the twelve notes (even on instruments with multiple manuals) made it by far the most widespread. On a broader scale, large orchestras, instrumental ensembles, as well as various encounters between voices and instruments generally prescribed the need to maintain a consistent pitch height to uphold a degree of "compatibility". However, it is important to keep in mind that all of this comes at the cost of significantly shrinking music's access to the immense wealth of tonal subtlety able to be perceived and enjoyed by the ear.

³⁴ Indeed, small sets of notes seem to be characteristic of the basic modes of many musical cultures around the world, ranging (on average) between 5–12 within the octave. However, many cultures likewise recognise various extended systems of inflection to their main pitch gamut(s), like the 22 *srutis* of Indian classical music, the 53 *komas* of Turkish *makam*, etc.

³⁵ This instrument, with 31 notes in the octave, made modulation and transposition possible through many keys while maintaining *nearly* pure triads, facilitating Vicentino's accompaniment of singers and instruments performing the increasingly chromatic and enharmonic music being composed at that time.

1.3 Extending just intonation: new musical expression

Ancient Greek practice held that adding an octave to a consonance (i.e. perfect fourth or fifth) remained consonant.³⁶ Walter Odington proposed that this property was not exclusive to the octave and that adding any of the consonances together may also produce a consonance, like the “double fifth” (*bis diapente*), i.e. major ninth: “to this end what is said, a consonance added to a consonance does not make a dissonance, it is a falsehood that a fifth added to a fifth makes a dissonance.”³⁷ However, he held that such “consonances” greater than an octave were not equivalent to their octave-reduced counterparts (e.g. the major ninth and major second), but possessed their own distinct qualities. This distinction is maintained in the practice of figured bass, in which figures 2 and 9 imply different contrapuntal resolutions. Giuseppe Tartini extended the consonances to include the minor seventh tuned as the seventh partial of the harmonic series alongside other proportions including 7, writing they are “easy to intone on the violin...[and] desired by harmonic nature.”³⁸ He ever proposed progressions in which the minor seventh—treated as a consonance—might rise melodically, in defiance of the standard rules of counterpoint.³⁹ In the last century, an increasing number of composers have focused their attention on a deeper examination of sound, harmony, and perception than ever before, daring to include intervals between many more partials of the harmonic series in their works.⁴⁰

One of the consequences of extending just intonation in this way is the increasing variety of intervals of *different sizes*. Of course, microtonality may also manifest itself in any number of other ways, ranging from essentially chance-based intonations to collections of *irrationally* related tones generated by dividing a base interval (historically the octave) into some number of equal parts. This approach to pixelating frequency space through equal divisions of the octave was championed by Iván Wyschnegradsky in his book *La Loi de la Pansonorité*⁴¹ and continues to be integral to many recent and current compositional practices, whether as full gamut or as subsets thereof, manifesting in the work of composers like Ezra Simms, Easley Blackwood, and Georg Friedrich Haas. However, my investigation in this essay will be limited

³⁶ Tenney, *A History of Consonance and Dissonance*, 11.

³⁷ “...unde ad hoc quod dicitur, consonum adveniens consono non facit dissonum, falsum est quod diapente adveniens diapente facit dissonum.” Odington, *Summa de Speculatione Musicae*, 14:72.

³⁸ “...di facilissima intonazione sopra il Violino...voluto dalla natura armonica...” Giuseppe Tartini, *Trattato Di Musica: Secondo La Vera Scienza Dell’armonia* (Padua: Giovanni Manfrè, 1754), 18.

³⁹ Sabat and Nicholson, *Fundamental Principles of Just Intonation and Microtonal Composition*, 15.

⁴⁰ Cf. E. Hamilton, H. Partch, B. Johnston, L. M. Young, J. Tenney, P. Oliveros, G. Grisey, W. Carlos, K. Gann, W. von Schweinitz, M. Sabat, C. Lamb, and many others.

⁴¹ Iván Wyschnegradsky, *La Loi de La Pansonorité*, ed. Franck Jedrzejewski and Pascal Criton (Paris: Contrechamps, 1996).

to approaches to composing music in just intonation—in particular, music that deliberately pushes the experience of harmonic sound *beyond* the fifth partial of the harmonic series, what Ben Johnston called *extended just intonation*. I examine an aesthetic angle that brings into question the relationship between tuneability and *how small* such so-called “small whole number proportions” of frequencies might become before losing all musical intelligibility (if ever).

An invaluable tool to visualise the complex network of relationships underlying extended just intonation will be James Tenney’s concept of “Harmonic Space”. Influenced by John Cage’s multidimensional “total sound-space, the limits of which are ear-determined only ... [with determinants such as] frequency or pitch, amplitude or loudness, overtone structure or timbre, duration, and morphology,” Tenney’s breakthrough was the realisation that “frequency or pitch” was itself a “multidimensional” phenomenon.⁴² If pitch were *only* a single-dimensional continuum (i.e. pitch height), then there would be, for example, no meaningful representation of the phenomenon of “octave equivalence”. Instead, the number of dimensions and depth of movement in each dimension (Harmonic Space coordinates) correspond to the prime bases and prime exponents of a ratio (or entire tuning, or piece, etc.). For instance, the isolated ratio 12/11 whose prime factorisation is $2^2 \times 3^1 \times 11^{-1}$, exists within at least a 3-dimensional Harmonic Space with axes corresponding to prime bases (2,3,11) and Harmonic Space coordinates (2,1,-1).

In the next chapter, I will investigate some approaches to composing in extended just intonation that has been influential on my own composing. In particular, I will look at Harry Partch’s system of *Monophony* and how he uses *Tonality Flux* to contrapuntally link distantly related just intonation chords. I will use his chamber work *Dark Brother* as a case study, which highlights many of the aesthetic strengths of his system, but also raises certain questions about perception. This will be followed by a supporting examination of extended just intonation from the point of view of the harmonic series and timbre. Various practices will be considered, focussing on Catherine Lamb’s string quartet *divisio spiralis*. In ch. 3, I propose a novel kind of “adaptive just intonation” that takes advantage of perceptual malleability without needing to introduce arbitrary tempering. In the same way that our eyes may deceive our senses in a 3-dimensional physical world, auditory and especially musical *perspective* at a given moment can play with our sense of “position” within Harmonic Space. I will explore practical as well as conceptual precedents in the repertoire, and offer analyses of some of the techniques I have used in my own compositions.

⁴² James Tenney, *From Scratch: Writings in Music Theory*, ed. Larry Polansky et al. (Illinois: University of Illinois Press, 2015), 294.

Chapter 2: Case Studies

In this chapter, I present various case studies examining just intonation from two perspectives. The first perspective, which is the primary investigation of this essay, concerns itself with an intervallic conception of just intonation. Specifically, sec. 2.1 centres around Harry Partch's technique of *Otonalities* and *Utonalities* interacting through *Tonality Flux*: close intervallic proximities bridging microtonal chordal structures. An analysis of Partch's 1943 composition *Dark Brother*, one of his earliest compositions to use this technique extensively, is provided (sec. 2.1.3) in the context of his 43-tone musical system and greater aesthetic interests. In sec. 2.2, I examine some further approaches to just intonation composition from the perspective of the extended harmonic series and spectral interaction in acoustic sounds. Recent works and practices from composers La Monte Young, Éliane Radigue, Ellen Fullman, and Catherine Lamb are considered, with a focus on a complementary analysis (sec. 2.2.1) of shifting modalities and neighbouring partials in the first third of Lamb's string quartet *divisio spiralis* (2019).

2.1 Tonality Flux: Harry Partch's *Dark Brother* (1943)

2.1.1 Thomas Wolfe: *God's Lonely Man*

Harry Partch was undeniably one of the most unique and experimental American musicians of the twentieth century. He spent his childhood in the Arizona desert, where he grew up with the sound of traditional Mexican and Yaqui Indian music in the air. His mother sang him Chinese melodies with their original Mandarin texts and encouraged him to study music. He learned to read music and to play several instruments including violin, mandolin, and piano. In early life, he acquired a growing dissatisfaction with formal institutions and a scepticism of

the mainstream musical establishment, opting instead for self-study through reading, composing, and experimentation.

After borrowing Hermann von Helmholtz's *On the Sensations of Tone as a Physiological Basis for the Theory of Music* from the Sacramento Public Library in 1925, his accumulating "doubts and ideas achieved some small resolution, and I [Partch] began to take wing."⁴³ Helmholtz's detailed study of acoustics and just intonation, including physical descriptions of consonance and dissonance, was a revelation. Within a year he had composed a string quartet in just intonation and by 1927 had completed the first of many drafts of what would eventually become his book, *Genesis of a Music*, in which he describes his method of working with 43 pitches per octave expressed as ratios, measuring intervals from a single reference, ♮G. Partch called this system "Monophony" and began building his own instruments based on ancient, contemporary, and completely original designs to play in this tonal framework: pump organs (called "Chromelodeons"), kitharas, marimbas, guitars, qanuns—the list is extensive.

His wayward and somewhat recalcitrant lifestyle often led him to rely on friends and acquaintances for room and board. In the fall of 1942, Partch moved to New York State to stay with mathematician Donald Flanders and his family, who had offered him their attic as a temporary studio for a few weeks. During his first stay at their farm a decade earlier, the family had developed a fondness for Partch and were sympathetic to his musical interests.⁴⁴ In the Flanders' attic, Partch was able to set up his instruments and focus his energies on composition as well as his second attempt at applying for a Guggenheim Fellowship. Outside of his artistic work, he would on occasion also participate in social gatherings hosted by the Flanders family, whose circle of friends included writer Thomas Wolfe's editor, Edward Aswell. If Partch and Aswell had indeed crossed paths in this period, it is not unimaginable, as Bob Gilmore suggests, that Aswell may have introduced Partch to the recently deceased Wolfe's writings.⁴⁵ In particular, one of Wolfe's essays titled *God's Lonely Man* had a lasting effect on the composer as it became the basis of a new composition: *Dark Brother*.

God's Lonely Man—or *The Anatomy of Loneliness* as it was later re-titled in a slightly abridged version for the magazine *The American Mercury*—is a concise distillation of the primary thematic thread that runs through Wolfe's entire body of work: loneliness. For Wolfe, loneliness "is the central and inevitable fact of human existence,"⁴⁶ and represents the particular emotional condition that unites all people. For this reason, loneliness should

⁴³ Harry Partch, *Genesis of a Music*, 2nd ed. (New York: Da Capo Press, 1974), vii.

⁴⁴ Bob Gilmore, *Harry Partch: A Biography* (New Haven & London: Yale University Press, 1998), 103.

⁴⁵ Gilmore, *Harry Partch*, 142.

⁴⁶ Thomas Wolfe, "God's Lonely Man," in *The Hills Beyond*, ed. Edward C. Aswell (New York & Toronto: The New American Library, 1941), 146.

actually be a source of joy rather than of despair because “the lonely man ... is invariably the man who loves life dearly—which is to say, the joyful man.”⁴⁷ Gérald Préher notes, “for Wolfe, human emotion is born out of loss; that is why love and loneliness are closely related.”⁴⁸ Humanity’s recognition and fear of loneliness and the certainty of death are the very things that heighten and give meaning to the experiences of joy, ecstasy, and love—which, for many, are the very aspects of life that afford it its preciousness.

It is not difficult to understand the connection Partch must have felt to a text like *God’s Lonely Man*. By 1942, he had become accustomed to an almost chronic sense of loneliness that accompanied his on-and-off wanderings around the United States. In *Bitter Music*, his diary chronicling life as a homeless man during the Depression, Partch often comments on the lonely, at times hostile conditions he had to endure. After arriving at a work camp south of Sacramento with two hundred other hobos in 1935, he comments,

I try to be friendly, but no one even looks at me, let alone answers me when I venture to speak.... My simple “hellos” do not penetrate their dream worlds. Vacant stares say: “How dare you try to insinuate yourself into this consciousness?” I decide to insinuate myself no more.⁴⁹

Throughout his artistic career, Partch—like Wolfe—would return time and time again to the theme of loneliness and its cold and bitter reality. Loneliness was inseparable from his own wayward lifestyle. Perhaps Partch was able to find some degree of consolation in Wolfe’s ultimately positive outlook on isolation in *God’s Lonely Man*, inspiring him to set the concluding two paragraphs as a sort of manifesto in *Dark Brother*.

But the old refusals drop away, the old avowals stand—and we who were dead have risen, we who were lost are found again, and we who sold the talent, the passion, and belief of youth into the keeping of the fleshless dead, until our hearts were corrupted, our talent wasted, and our hope gone, have won our lives back bloodily, in solitude and darkness; and we know that things will be for us as they have been, and we see again, as we saw once, the image of the shining city. Far flung, and blazing into tiers of jeweled lights, it burns forever in our vision as we walk the Bridge, and strong tides are bound round it, and the great ships call. And we walk the Bridge, always we walk the Bridge alone with you, stern friend, the one to whom we speak, who never failed us. Hear:

Loneliness forever and the earth again! Dark brother and stern friend, immortal face of darkness and of night, with whom the half part of my life was spent, and with whom I shall abide now till my death forever—what is there for me to fear as long as

⁴⁷ Wolfe, “God’s Lonely Man,” 149.

⁴⁸ Gérald Préher, “A Cosmos of His Own: Loss, Ghosts, and Loneliness in Thomas Wolfe’s Fiction,” *The Thomas Wolfe Review* Vol. 35, no. 1 & 2 (2011): 25.

⁴⁹ Harry Partch, *Bitter Music: Collected Journals, Essays, Introductions, and Librettos*, ed. Thomas McGeary (Urbana & Chicago: University of Illinois Press, 1991), 16.

you are with me? Heroic friend, blood-brother of my life, dark face—have we not gone together down a million ways, have we not coursed together the great and furious avenues of night, have we not crossed the stormy seas alone, and known strange lands, and come again to walk the continent of night and listen to the silence of the earth? Have we not been brave and glorious when we were together, friend? Have we not known triumph, joy, and glory on this earth—and will it not be again with me as it was then, if you come back to me? Come to me, brother, in the watches of the night. Come to me in the secret and most silent heart of darkness. Come to me as you always came, bringing to me again the old invincible strength, the deathless hope, the triumphant joy and confidence that will storm the earth again.⁵⁰

In addition to the typological delineation into two paragraphs,⁵¹ one can distinguish two distinct thematic focusses in these concluding lines to Wolfe's essay. The first paragraph begins looking inwardly, speaking of the "we" that comprises the community of all people who are or have ever been lonely (including the author in particular). Almost as if delivering a sermon, Wolfe proclaims the inherent redemption in our experience of loneliness, summarising many of the points he explores in the preceding text: "we who were lost are found again, and we ... have won our lives back bloodily, in solitude and darkness." By the end of the paragraph, the focus gradually turns toward the "stern friend" who "never failed us"—i.e. toward a personification of loneliness itself. The second paragraph is more personal in tone as Wolfe, abandoning the "we" from the previous paragraph, begins addressing loneliness directly in terms of his own experience. Through thick and thin, Wolfe could always depend on his "Dark Brother" to stand at his side: "have we not gone together down a million ways?" As the paragraph progresses, clauses become shorter and more paratactic and the language acquires a more rhythmical flow, aided by the repetitions of "Come to me" beginning the final three sentences. The overall effect is one of increasing tension until the cathartic closing declaration of "triumphant joy and confidence"—the ultimate consequence of loneliness according to Wolfe's view.

While the bulk of the essay is stylistically written in prose and somewhat rhetorical in tone, there is a marked shift toward the poetic in these last two paragraphs. Wolfe crafts long sentences built up from strings of basic statements, which impart a sense of declamatory jubilation and even hysteria. This exultant, "run-on" style would become a characteristic of the subsequent generation of Beat writers such as Jack Kerouac, for whom Wolfe and his commentary on American mores were creatively as well as conceptually influential. In

⁵⁰ Wolfe, "God's Lonely Man," 154.

⁵¹ Curiously, Wolfe creates a short third paragraph consisting of the last two sentences in his subsequent republication titled *The Anatomy of Loneliness*. This was later reproduced in Scribner's anthology of his complete short stories. The reason for this retroactive re-partitioning of the text is unclear, especially since it appears to separate a single thematic unit—namely the last three phrases, which beckon loneliness to "Come to me [Wolfe]".

particular, a similar approach to style can be observed in Allen Ginsberg's poem *Howl*, published in 1956. Ginsberg experiments with stringing together breath-length outcries often consisting of short, fragmented statements that lend an element of theatricality. The dramatic nature of these "howls" becomes particularly salient when the text is actually read aloud, as in Ginsberg's own recordings of the poem. Consider the following line from Part I:

who sang out of their windows in despair, fell out of the subway window, jumped in the filthy Passaic, leaped on negroes, cried all over the street, danced on broken wineglasses barefoot smashed phonograph records of nostalgic European 1930s German jazz finished the whiskey and threw up groaning into the bloody toilet, moans in their ears and the blast of colossal steamwhistles⁵²

Furthermore, the use of repeated words and phrases to begin extended sequences of lines—such as "who" in Part I, "Moloch" in Part II, and "I'm with you in Rockland" in Part III—is reminiscent of Wolfe's incantations of "Come to me".

In experiencing Ginsberg's readings, there is no question that the act of "performing" *Howl* clarifies and indeed heightens its innate drama. Surely, Partch must have recognised a similar potential in Wolfe's text. The dramatic nature of the spoken word was particularly attractive to Partch, who, a decade earlier, had articulated in *Bitter Music* some of his motivations for treating text as a musical material with such directness. He explains that he aims to

give myself, and others, a good basis for a new and great music of the people ... And that's why I work with words, because they are the commonest medium of creative expression. And words *are* music. Spoken words.⁵³

Partch scored *Dark Brother* for voice accompanied by a small ensemble of home-made instruments: Adapted Viola, Chromelodeon, Kithara, and Bass Marimba (details below).

2.1.2 Analytic preparations

The instruments and their notation: tablature and ratios

Anyone who desires to gain insight into Partch's music, whether for performance or analysis, is quickly faced with a formidable obstacle: notation. Partch's scores rarely offer notational information about how they ought to *actually* sound. Instead, they favour a combination of staff-based tablature specific to each of his home-made instruments and numeric ratios. The tablature fulfils a purely practical role, providing physical instructions to the performer. For the most part, Partch retains the traditional five-line staff but notes on lines and in spaces

⁵² Allen Ginsberg, "Howl," in *Howl and Other Poems* (San Francisco: City Lights Books, 1956), 17.

⁵³ Partch, *Bitter Music*, 12.

acquire new, instrument-specific significance. For many of his instruments, staff markings indicate which part of the instrument to play or which string(s) within a set to sound.

Most of the instruments in *Dark Brother* are notated in the manuscript with this type of tablature. The “Kithara I”, comprising twelve hexachords of strings tuned to Partch’s twelve Primary Tonalities (discussed in sec. 2.1.3), are notated employing only the staff-spaces. Accompanied by a number between 1 and 12 indicating *which* hexachord ought to be played, notes in the six staff-spaces represent the six strings from nearest to the performer (bottom of the staff) to farthest (top of the staff). An example of the Kithara’s⁵⁴ tablature is given in fig. 7 and again in fig. 9. Similarly, Partch notates the eleven pitches of his “Bass Marimba”, a massive wooden marimba tuned in the range of the cello’s open strings, on all lines and spaces of the staff, from the “space” below the lowest line to the “space” above the highest line (see fig. 8). Visually, this linearity gives the impression that the tonal gamut might consist of similarly spaced pitches or melodic steps. However, in actuality, the Bass Marimba’s range spans nearly two octaves, consisting of interval steps anywhere from a quarter-tone (33:32, ca. 53 cents) to a wide diminished fourth (11:16, ca. 418 cents).

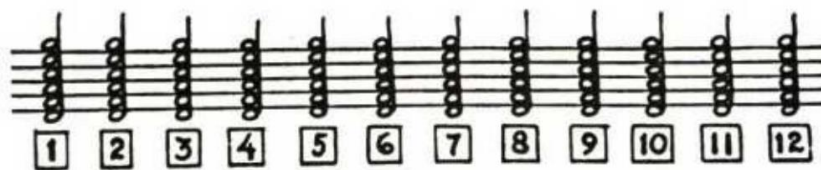


Figure 7: Partch’s Kithara tablature diagram from *Genesis of a Music*.

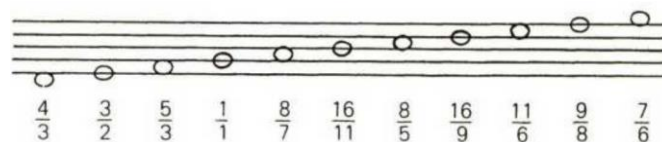


Figure 8: Partch’s Bass Marimba tablature diagram from *Genesis of a Music*. The pitch 1/1 is equivalent to the tuning of a cello’s G string.

Partch’s “Chromelodeon I” notation is another example of tablature, making use of the traditional staff notation only as an indication of which keys to press (fig. 9). For the most part, the sounding pitches are, unsurprisingly, drastically unrelated to the typical 12-tone keyboard tuning. In Partch’s 43-tone scale, the Chromelodeon⁵⁵ has a reduced range of just

⁵⁴ In reality, Partch built multiple Kitharas, Kithara I being his first. In the remainder of this essay, any reference to simply “Kithara” refers to Kithara I, as was used in *Dark Brother*.

⁵⁵ Similar to the Kitharas, Partch actually built two Chromelodeons. Any subsequent reference to simply “Chromelodeon” therefore refers to Chromelodeon I, as was used in *Dark Brother*.

over an octave and a half because 43 keys are required to produce a sounding octave. Additionally, like any small pump organ, the Chromelodeon has a series of stops that divide the keyboard between the keys E and F below middle C. The A^L and A^R stops activate reeds for the regular registration of the instrument, like a traditional organ's 8' stop. The Z stop activates a row of reeds tuned an octave lower in the left half of the keyboard (16') and the X stop activates a row of reeds tuned an octave higher in the right half (4'). The most interesting stop is probably the $\frac{6}{5}$ stop, which is the organ's octave coupler affecting the right half of the keyboard: rather than introducing a new row of reeds, the octave coupler (under typical conditions) creates the effect of a 4' stop by mechanically coupling the key an octave above the key actually being played. However, an octave on the Chromelodeon's keyboard only produces an interval of 12 steps in Partch's 43-tone system, thus, on average, this coupling produces approximately a minor third (frequency ratio $\frac{6}{5}$). Partch primarily uses this stop as a special timbral effect, especially in combination with arpeggios and trills.

Interestingly, after initially notating the intoning voice solely in ratios (as he did for the Adapted Viola, see below), he quickly switched to representing it in Chromelodeon notation. This was presumably to allow the Chromelodeon to function as a clear point of reference for the voice during the process of learning as well as in performing his compositions. Partch notated the voice in *Dark Brother* in Chromelodeon notation.

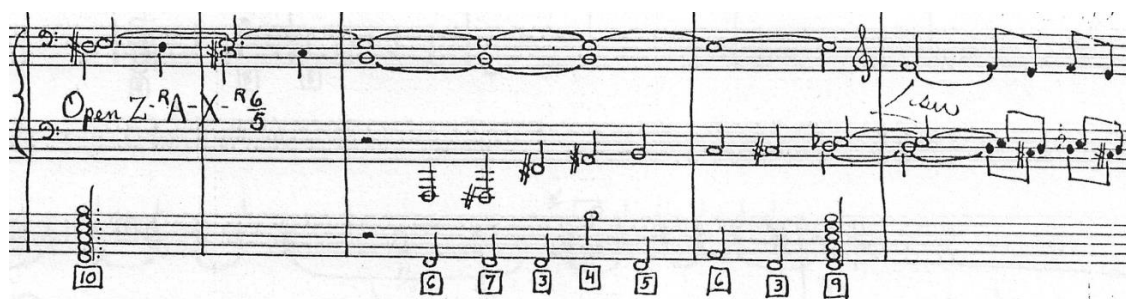


Figure 9: The opening of *Dark Brother* (manuscript), showing tablature for Chromelodeon I (grand staff) and Kithara I (lowest staff).

For the Adapted Viola—Partch's very first home-made instrument, consisting of a cello fingerboard attached to the body of a viola—he preferred notating tones as a string of ratios in relation to the note $\sharp G$ rather than as notes on a staff (see fig. 10). The three octaves of the instrument's normal playing range are each indicated by a numerical "clef" positioned on the staff before a musical passage. Tones are always indicated in "normalised" form a/b where $2b \geq a \geq b \geq 1$. In more musical terms, tones are represented as octave-reduced ratios with respect to the nearest $\sharp G$ below them. While this technique removes any ambiguity regarding pitch—albeit in a graphically unusual and abstract way—it is decidedly impractical. Given

any two arbitrary pitches expressed as ratios in relation to $\flat G$ it is not necessarily obvious which will be the higher or lower, and, by extension, whether the melodic interval between them moves upward or downward. Since this information is vital to the mechanical aspect of producing tones, particularly on a stringed instrument where no tones (beyond the open strings) are fixed, Partch's ratio notation is, curiously, in diametric opposition to his pragmatic staff-based tablature.

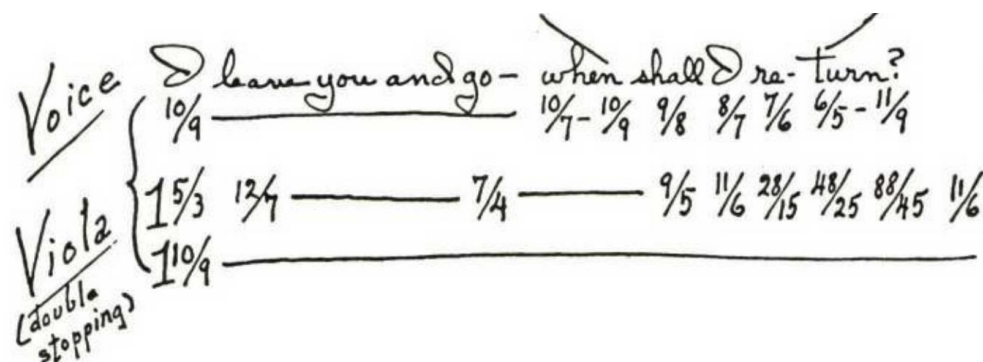


Figure 10: A passage from Partch's *Seventeen Lyrics by Li Po* showing his ratio notation—in these songs, the voice is also notated in ratios.

A notational solution: transcription into HEJI

A solution somewhere between Partch's two methods would be useful and probably necessary to realise any in-depth analysis of his music. Fortunately, several extended just intonation accidental systems have been devised over the past fifty years. Among these, the most widely adopted notations are the system of Ben Johnston, the Helmholtz-Ellis Just Intonation Pitch Notation (HEJI), and Sagittal Notation. Naturally, any one of these accidental systems comes with its own caveats, which are in no way completely divorced from those already imposed by Partch's own notational choices. They do, however, enable the graphical representation of just intonation in a precise and reasonably familiar-looking *musical* way. Johnston's notation and HEJI both retain the traditional sharps and flats, as do modified variants of Sagittal. In these systems, rough estimates of the desired pitch height are more intuitive for most musicians, even for those who are relatively new to both reading and interpreting an explicitly microtonal notation.

The first step in decoding Partch's manuscript for *Dark Brother* was to gather specifics about the various tablatures (described above). The tuning details (in ratios) for these instruments were published in *Genesis of a Music*. The fact that Partch often modified the tunings of his instruments certainly does not simplify matters and one can easily be led astray by differences between the two editions of his book. Fortunately, however, Partch had only one tuning for

Chromelodeon I and the Bass Marimba, and he includes the helpful annotation in the second edition of *Genesis of a Music* that “the tuning [of Kithara I] given in the first edition has not been used since 1952.”⁵⁶ Therefore, it can safely be assumed that *Dark Brother*, having been composed between 1942–1943, utilises the first edition tuning of Kithara I.⁵⁷ Tuning charts for Kithara I and Chromelodeon I are given in appendix B (the Bass Marimba’s pitches can easily be extrapolated from fig. 8, so I did not feel it necessary to make its own separate tuning chart).

With this information, Partch’s manuscript could be transcribed into HEJI notation. This choice was made for four reasons. Firstly, HEJI is the notation I use in my own composing and is, therefore, familiar to me. This reduced the risk of introducing errors (an unfortunate though common trap when working with Ben Johnston’s notation) while facilitating the overall process of analysis. Secondly, of the three aforementioned major just intonation notations, HEJI is ostensibly the least ambiguous graphically as well as the least unusual looking. While it functions similarly to the others in that each prime “family” is represented by a unique symbol (see below), HEJI’s symbol set has the most variation and is visually robust. This ensures that symbols do not get misinterpreted as others. Equally, certain symbols do not “disappear” when they interact with the staff itself, something that Johnston’s notation, unfortunately, suffers from with its strings of small pluses, minuses, and upside-down numbers. Thirdly, the following two chapters of this essay investigate just intonation compositions by Catherine Lamb, myself, and other composers, all of whom use HEJI in their practices. For the sake of continuity throughout this larger investigation, it is convenient for *Dark Brother* to also be notated in the same system of JI accidentals. And finally, I wanted to prepare a kind of “performance edition” for this little known masterpiece in the hope that, as familiarity with HEJI continues to reach an ever-broader population of musicians and the current revival of interest in Partch’s work grows (e.g. Musikfabrik in Cologne), it might find a place more often on concert programs. The complete transcription is provided at the end of this essay in appendix C.

⁵⁶ Partch, *Genesis of a Music*, 223.

⁵⁷ Partch was eventually dissatisfied with the initial tuning, lamenting that the “close bass intervals produce a muddy effect.” If his later tuning, the intervals are more spread out and the strings were shortened, bringing the instrument into a more alto register.

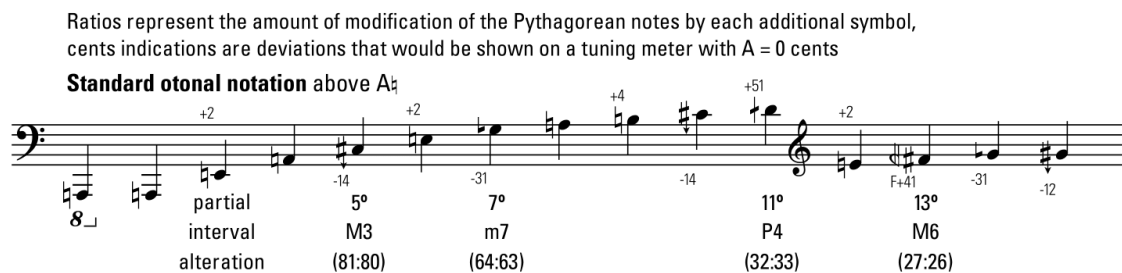


Figure 11: The first 15 partials of the harmonic series on A₄ notated in HEJI. Many of the notation's characteristic symbols can be seen. For Partch's music, only the arrow for 5° (which attached to its Pythagorean accidental), the "Tartini" symbol for 7°, and the classic quarter-tone symbols for 11°, in combination with the Pythagorean sharps and flats are required to be notated with staff notation.

In the early 2000s, Marc Sabat and Wolfgang von Schweinitz developed HEJI as a tool to provide "visually distinctive 'logos' distinguishing [prime] 'families' of natural intervals based on the harmonic series."⁵⁸ From its inception, the aim was to create new signs derived from the traditional staff notation. The basic unaltered pitches—i.e. those notated with the traditional sharps and flats ♭, ♮, ♯, etc.—are Pythagorean, representing the 3-dimension of Harmonic Space. To notate other prime dimensions (and mixtures thereof), the Pythagorean notes must be altered by various commas, each of which is indicated by its own symbol that is either attached (in the case of the 5-dimension) or placed before the Pythagorean accidental it alters (for all others). Fig. 11 presents the harmonic series through the fifteenth partial of A₄ to illustrate how the notation works in practice. For example, the 5-dimension is characterised by an alteration by the syntonic comma 81:80, which is indicated by an arrow attached to the Pythagorean accidental, e.g. ♯C for 5th partial. However, since any pitch may be potentially raised *or* lowered by a comma, each symbol has two corresponding forms, which are typically just mirror images along the horizontal axis.⁵⁹ Consider how, in the hypothetical subharmonic series of A₄, the 5th subharmonic would be logically raised, ♮F. A complete guide to HEJI is provided in appendix A.

Note that in conjunction with the staff notation I will also uphold a "ratio notation" convention for the remainder of this essay: for an interval of frequencies *a* with respect to *b* that are sounding simultaneously, I will write them in quotient form, i.e. *a/b*, where the numerator is with respect to the denominator—on the other hand, if they are sequential, a so-called "melodic interval", I will write them in ratio form, i.e. *a:b*, where the frequency *a*

⁵⁸ Marc Sabat, "The Extended Helmholtz-Ellis JI Pitch Notation," 2005, 4, <http://www.marcsabat.com/pdfs/notation.pdf>.

⁵⁹ Notable exceptions are the symbols for the 11- and 13-dimensions, which are based on historical accidental for quarter- and third-tones (11: ♯ and ♮, and 13: ♯ and ♮).

changes to frequency b after some arbitrary amount of elapsed time. Chords are written in the form of a proportion, i.e. $a:b:c:\dots$

Streamlining with software: the Harmonic Space Calculator

A final step in streamlining the process of converting ratios into their HEJI accidentals was to create a browser-based JavaScript *Harmonic Space Calculator*.⁶⁰ Given any input ratio, this program displays the correct HEJI notation along with other information such as frequency, cent deviation from the nearest equal-tempered note, Harmonic Space coordinates, and notation software specific pitch bend information. Another useful feature is its *melodic step* function, which calculates the ratio *between* two ratio inputs, providing information such as cents, frequency difference, etc. This, as will be seen in the next section, is especially pertinent to Partch's musical style, which often involves glissando-like microtonal melodies and voice leading.

The functionality of the *Harmonic Space Calculator*, however, was intentionally designed to be flexible. Though Partch's music is 11-limit, my program allows for staff notation of prime dimensions up to the 47th and for cent deviation and frequency calculation for *any* prime-limit able to be handled by the computer's hardware.⁶¹ The choice of note and tuning for the reference (1/1) may be explicitly defined (for Partch, this is $\flat G$ tuned to 392 Hz) and the conversion of information may flow in either direction: i.e. from ratio to notation *or* from notation to ratio. Therefore, the *Harmonic Space Calculator* is an equally useful tool for the composition of new music in extended just intonation and has been invaluable in my own practice (described in the next chapter). Fig. 12 shows a screenshot of the Calculator in action.

⁶⁰ Thomas Nicholson, "Helmholtz-Ellis 31-Limit Harmonic Space Calculator," 2018, <http://www.plainsound.de/HEJI/>.

⁶¹ For anyone interested in such pedantics, a 64-bit system storing integers with 53-bit precision (maximum value $2^{53}-1$) can access a maximum prime dimension of $2^{53}-111 =$ the 9007199254740881-dimension. Interestingly, normalising this prime to $9007199254740881/4503599627370496$ essentially yields an octave (1199.9999999997864 cents). Using Gauss's prime-counting function, $\pi(x) = x/\ln(x)$, which approximates the number of primes less than or equal to a real number x , we find that the total number of prime dimensions accessible to the *Harmonic Space Calculator* to be ca. 2.5×10^{14} . Nevertheless, the calculator only specifies staff notation for the first 14 prime dimensions.

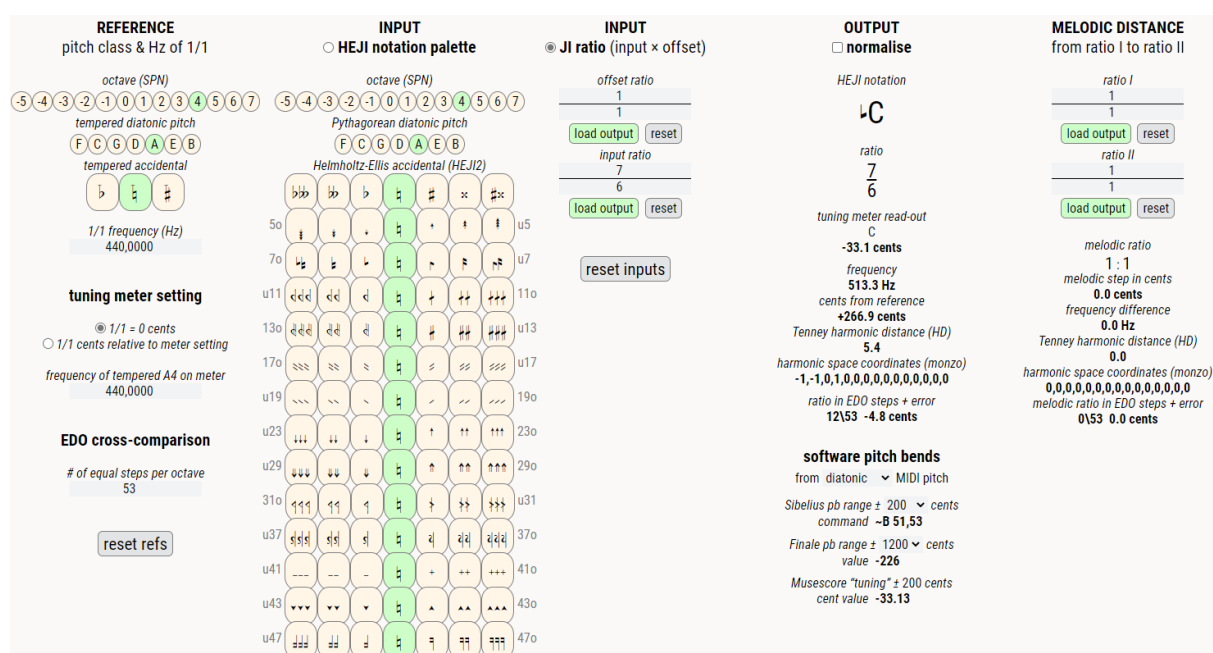


Figure 12: Screenshot of the Harmonic Space Calculator being used to calculate practical information about the pitch 7/6 above a reference of A 440 Hz.

2.1.3 Musical analysis

General description of the form and motivic elements

The characteristics of spoken words—how they are intoned, how they group together, how they suggest larger forms (sentences, paragraphs)—underpin Partch’s unique compositional voice, both in terms of structure and expression. In particular, he was attracted to the potential musical qualities of a slightly more dramatic or, perhaps, theatrical expression of spoken text while retaining its inherent intelligibility. Even how he himself spoke might be described as possessing a certain theatricality, as evidenced by his many recorded demonstrations and commentaries.⁶²

Dark Brother’s musical form is clearly and intimately linked to the linguistic structures of Wolfe’s text. Bookended by a brief instrumental introduction and coda, Partch builds the main body of his composition in two distinct sections, mirroring the two paragraphs of the text. He imposes no repetitions of words or phrases, allowing Wolfe’s text to unfold linearly and organically over the duration of the piece as if read aloud or recited in a play. The formal plan of *Dark Brother* is summarised in tbl. 2. All references to measure numbers in this essay refer to my transcription (appendix C).

⁶² Refer especially to *Enclosure II* from Innova Records, which is a re-release of a number of Partch’s self-made recordings and demonstrations.

Section		Measures	Details
Introduction		1–11	Instrumental
	<i>Transition</i>	12–13	
A-Section	<i>Part 1</i>	13–44	First paragraph (Wolfe)
	<i>Transition</i>	44–58	
	<i>Part 2</i>	59–75	
B-Section	<i>Part 1</i>	77–117	Second paragraph (Wolfe)
	<i>Part 2</i>	118–135	
“Coda”		136	Instrumental

Table 2: Formal outline of *Dark Brother*.

The image displays the opening phrase of the piece 'Dark Brother' for three instruments: Chromelodeon I, Kithara I, and Bass Marimba. The score is written in bass clef for the first three staves and treble clef for the fourth staff (Ch I). The key signature has two sharps (F# and C#). The Chromelodeon I part includes a box labeled 'Open Z-A^R-X⁶₅'. The Kithara I part is marked '(first tuning)'. The Bass Marimba part is marked 'Bass Marimba'. The Ch I part is marked 'Ch I'. The score shows measures 1 through 11, with a double bar line and a repeat sign at the end of measure 11.

Figure 13: Opening phrase of *Dark Brother*.

In fig. 13, the piece opens in the low register with Chromelodeon, Kithara, and Bass Marimba, which present two similar phrases (mm. 1–5, 7–11). The first phrase is initiated with a single, grunt-like “impulse” played by the Kithara and Bass Marimba, out of which the

Chromelodeon presents a melodic line in compact microtonal steps against a sustained $\flat B$ drone. First descending from $\flat B$ in mm. 1–2 with a gently lilting rhythm, the Chromelodeon leaps down a perfect fourth, prompting an ascent in half notes in mm. 3–4 harmonised by Kithara and Bass Marimba. A second “impulse” at the end of m. 4 marks the beginning of the second half of the first phrase. Here, against low, beating dyads, the Chromelodeon adds its higher register and presents the main motivic phrase of the composition, which I have named the Rocking Gesture. This phrase contains two components: a gentle rocking back and forth of a narrow interval in eighth notes followed by a small leap upward and a descending continuation in half notes. A sudden silence at the end of the phrase—as if a speaker were interrupted mid-thought—joins the first phrase to the second.

The second phrase begins almost identically to the first with the same “impulse”, descending–ascending Chromelodeon figure, and harmonisation in the other instruments. The second part of the phrase expands on the continuation of the Rocking Gesture by inverting its melodic shape and widening the melodic steps, ending somewhat heroically on an insistent re-articulated quasi-B major triad. The two-part shape of this opening material (mm. 1–11) is reminiscent of a classical *period*.

Finally, a short transitional phrase (mm. 12–13) bridges the introduction with the A-section. In this transition, the Chromelodeon presents almost the most basic demonstration of Partch’s conception of *Tonality Flux*—essentially voice leading that reveals “subtleties of [T]onality relationships” through small microtonal steps.⁶³ In this case, a two-voice counterpoint consisting of inward contrary motion in half notes (m. 12, see fig. 14) coalesces on the note $\flat A$ (m. 13). This particular wedge-like example of elementary Tonality Flux reoccurs as a structural element later in the work, so I have simply named it the Wedge.



Figure 14: *The Wedge: a basic inward Tonality Flux culminating on the pitch $\flat A$.*

The beginning of the main body of the composition is signalled, once again, in m. 14 by a Kithara-Marimba “impulse” followed by the subtle entry of the Adapted Viola on $\flat A$, which seems to bloom out of the decay of the Chromelodeon’s same concluding tone. The short

⁶³ Partch, *Genesis of a Music*, 190.

ensuing instrumental passage (mm. 14–18), consisting of a gentle melody played by the Adapted Viola accompanied with delicate single tones in Kithara and Bass Marimba, immediately sets the intimate chamber character of the A-section. When the voice finally enters at the pick-up to m. 19, its supporting instrumentation is reduced to only Adapted Viola and Chromelodeon, providing *speech harmonisation*.

Partch builds small-scale passages that closely follow the linguistic structures of the text he is setting. Without exception, clause-length phrases intoned by the voice are followed by brief instrumental “responses”. These range from extended melodic fragments (e.g. mm. 22 and 26) to quick microtonal harmonic shifts (e.g. mm. 32 and 34), bridging phrases led by the voice. In clearly delimiting clauses in time, he highlights the paratactic quality innate to Wolfe’s text, itself constructed of long strings of clauses (see discussion in sec. 2.1.1).

The unfolding of the music is primarily through-composed and, occasionally, Partch makes use of “word painting” in the instruments to enhance and draw attention to aspects of the text. For instance, consider the almost naively rising Adapted Viola figure in m. 24, which imitates the line “and we who were dead have risen,” as well as the voice’s arrival in the following measure on ♮G—the “home” pitch, 1/1, of Partch’s entire harmonic system—when it intones “we who were lost are found again” (see fig. 15). Mostly, however, his focus is on capturing the voice’s dynamic intonation as a *by-product* of its correlation to potential emotional and expressive stimuli. Contours and melodic gestures follow and, in some sense, aim to synthesise the natural rise and fall of English speech (sometimes termed “cadence”). The instruments support the voice’s tones through extensive doubling and simple harmonisations, which encourage it to “lock into” the ever-shifting harmonies.

Figure 15: Word painting in. mm. 23–26. The rhythmic notation will be clarified in the next section.

In fig. 16, a terse sixteenth note “jolt” in the Adapted Viola, as well as the marked shift in texture (density) caused by the reintroduction of the Chromelodeon’s X and 65 stops, signal the start of a transitional passage in the middle of m. 44. Here—as well as in m. 48—the Chromelodeon begins to weave the Rocking Gesture into the musical stream. Any hierarchical boundary separating the role of the voice and the role of the accompanying instruments begins to blur and the previous order of *who influences whom* becomes less clearly defined. In both instances, the voice attempts to “sing along” to the motif initiated by the Chromelodeon, but quickly abandons its effort and returns to its more natural speech cadence (mm. 45 and 46). Further sixteenth note “jolts” follow in mm. 50–51: a fragmentation of the Rocking Gesture’s elaborated continuation presented in m. 11 of the introduction. The combination of all these elements underscores the increasing expressive and psychological intensity of the text, which culminates in the third and most complete instance of the Rocking Gesture on the word “Bridge” (end of m. 56). In this instance, the voice makes no further attempt to sing along with the Adapted Viola and Chromelodeon—perhaps left breathless by the preceding excitement and ever-rising register.

The musical score is divided into two systems. The first system covers measures 44 to 47, and the second system covers measures 48 to 52. The staves are labeled A Vla, V, and Ch I. The lyrics are: "and we see a - gain as we saw once," and "the im - age of the shin - ing cit - y. Far flung,". A box labeled "Open X-6" is present in the Ch I staff in measure 45. The score includes various musical notations such as notes, rests, and dynamic markings.

Figure 16: “Jolts” and Rocking Gesture in mm. 44–52.

Part 2 of the A-section begins in m. 59 and is characterised by a strong change in harmonic rhythm. The listener’s attention is dominated by the $\flat G$ pedal tone in the Chromelodeon, which, for the voice, becomes a point of strong gravitational stability. Short vocal intonations are bridged with persistent iterations of $\flat G$ by the Adapted Viola (mm. 60, 62, 64, 66) that are further emphasised by fermatas (e.g. in fig. 17). This harmonic focus on $\flat G$, Partch’s unity 1/1, may be a reflection of the narrator’s feeling of unity with Loneliness: “always we walk the bridge, alone with you, stern friend.” Two ascending melodies in the Chromelodeon’s left hand against the pedal $\flat G$ (mm. 70–71 and 72–73) accompany the closing clauses of the A-section, leading to the final outcry of “Hear” in m. 75. A recurrence of the Wedge in the Chromelodeon’s mid register (roughly an octave higher than in the introduction) serves as a beacon in m. 76, connecting the A- and B-sections.

61

A Vla

V

and the great ships call. And we walk the Bridge, al - ways we walk the Bridge

Ch I

Figure 17: \flat G pedal tone.

The first part of the B-section begins with an instrumental march-like interlude (mm. 77–78) that provides a stark contrast to the essentially “free form”, stream-of-consciousness flow of the A-section. From the beginning, an incessant quarter note pulsation is established by the Kithara and Bass Marimba. The ostinato affords an almost “primaeval” or perhaps “ritualistic” directness to the ensuing music. Over the course of the 20-beat-long m. 78, the Adapted Viola presents an extended folksong-like melody accompanied by virtuosic figurations and tremoli in the Chromelodeon, which create clouds and hazes of tones that enrobe the underlying march. This use of the instrument is characteristic of Partch’s writing for Chromelodeon, with similar effects featuring in other works such as *U.S. Highball* composed shortly after *Dark Brother*. In particular, Partch’s background as an accomplished pianist is evident in this section of *Dark Brother*, where the keyboardist is called upon to execute demanding figurations in the right hand while simultaneously reinforcing the many tones in the voice with the left hand (e.g. in fig. 18).

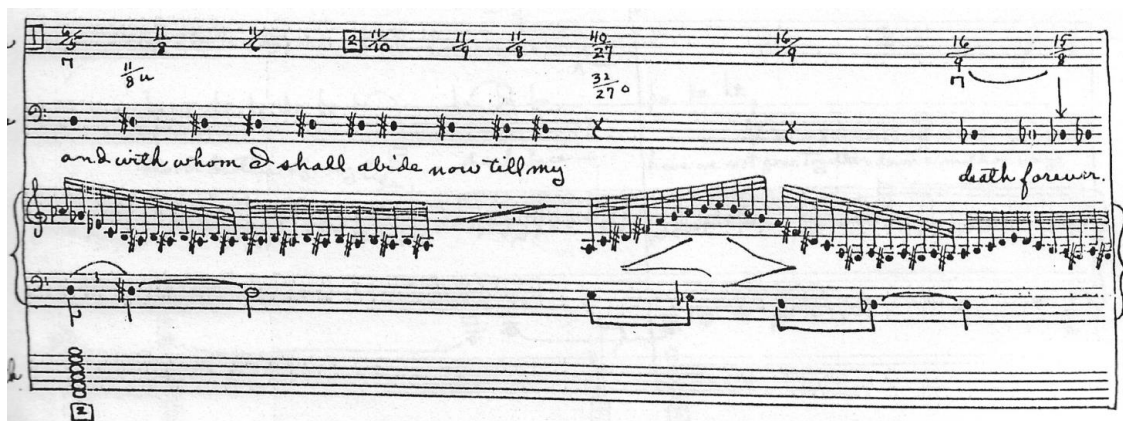


Figure 18: Virtuoso writing for the Chromelodeon (grand staff), here clearly seen in the instrument's tablature in the manuscript.

Once again, instrumental responses are inserted to bridge intonations by the voice. These have a much more decisive, even brutal effect compared to the fantasy-like insertions found in the A-section. Generally, these responses are more compressed and typically comprise a *short-long* rhythmic motif articulated by the Kithara and Bass Marimba, which I call the Knocking Gesture. In its initial instances (mm. 79–85), the “short” element of its characteristic rhythm is a quarter note, creating a strong link to the preceding march. After brief fragments of more overt references to the march in mm. 87–94, Partch reverts to the Knocking Gesture and begins to transform the duration of its “short” component. In m. 98, it is compressed to an eighth note and in mm. 100 and 102 it is further shorted to a single sixteenth note. This stretto proves to be more of a local phenomenon than a structural device—it does not “arrive” anywhere. From m. 104, the length of the “short” element returns to an eighth note. Nevertheless, Partch makes use of the Knocking Gesture’s simplicity as a perceptible basis for motivic experimentation and development to underline the increasingly hysterical nature of the text, occasionally leaving instruments out (e.g. both Kithara and Bass Marimba in m. 96; Kithara in m. 100) as well as adding a “stutter” (e.g. short-short-long in m. 114). While he focuses on such timbral variation (through instrumentation) and rhythmic transformation—which has the potential to be quite rich—Partch does not really explore the combinatorics too deeply, and the whole “development” passage passes by rather swiftly.

A short resuming of the instrumental march in m. 117 leads into the second part of the B-section. Parallels are drawn to part 2 of the preceding A-section. Ascending microtonal counterpoints in the Chromelodeon’s left hand, which bridge voice entries, are reminiscent of the ascending lines at the end of the A-section. A strong tendency toward the note $\flat B$ echoed back and forth between the Adapted Viola, voice, and Chromelodeon strengthens the impression of an underlying pedal tone as well. Partch examines the Harmonic Space

immediately around this tone by slightly raising and lowering the pitch—first raising it 32 cents to $\sharp B$ in m. 122, then lowering it 84 cents to $\natural A$ in m. 124. The return to $\flat B$ by the end of the measure unfolds in two steps comprising 63 cents (through Pythagorean $\flat B$) and 22 cents. Though a simple idea, this explicit probing of the tonal microcosm surrounding a single tone of focus—in a sense, discovering its gravity in different harmonic contexts—is arguably one of Partch’s most innovative contributions to music on a wider level. Though Partch’s work was likely unknown for the most part (in detail) to many of his contemporaries both in Europe and North America, the ripples of this particular technique may nevertheless be observed in numerous subsequent compositions and became the basis of Giacinto Scelsi’s so-called “monotonal” style.

After the voice’s final words in m. 134, *Dark Brother* concludes with an instrumental coda (fig. 19). The Chromelodeon trails off early in the passage, leaving only Adapted Viola, Kithara, and Bass Marimba to finish the work. The coda’s structure mirrors the introduction in an abridged form, consisting of two melodic phrases in the Adapted Viola derived from the Rocking Gesture accompanied by steady, folk-song-like strumming in the Kithara. Rhythmically, the Bass Marimba alternates between supporting the Adapted Viola melody and the Kithara accompaniment. Despite the text’s apparently heroic message, the low register, quiet dynamic, and sombre gait of the music brings the piece to a decisively dark, perhaps tragic close, cut off by a single, final reminder of the Knocking Gesture in solo Bass Marimba marked *pianissimo*.

136

A Vla

Ch I

Kth

B M

138

A Vla

Kth

B M

pianissimo

The musical score is presented in two systems. The first system, starting at measure 136, includes four staves: A Vla (bass clef), Ch I (treble clef), Kth (bass clef), and B M (bass clef). The second system, starting at measure 138, includes three staves: A Vla (bass clef), Kth (bass clef), and B M (bass clef). The music is in a key with two flats and a common time signature. The first system shows a complex texture with many chords and moving lines. The second system shows a more sparse texture with sustained chords and moving lines. The B M part in the second system ends with a 'pianissimo' marking.

Figure 19: Instrumental Coda to Dark Brother.

Rhythmic characteristics

In *Genesis of a Music*, Partch writes:

Although Westerners have used basic rhythms of 5 and 7 to good effect, the field of exploration in the more complex forms of 5 and 7, the simple impulses of 11 and 13, and the rhythm of speech, is fabulously extensive.⁶⁴

He chooses to address these “fields of exploration” in his work by employing two rhythmic paradigms with corresponding notations, both of which appear in *Dark Brother*.

1. Standard rhythmic notation is used for notating instrumental passages that have a *conventional sense of metre* (i.e. do not follow speech).
2. The *natural rhythm of speech* is notated without note stems and is logically reserved for passages with voice (and accompaniment). Solid noteheads indicate the duration of a single syllable and hollow noteheads indicate a duration spanning two or more syllables.

In *Dark Brother*, these rhythmic paradigms alternate depending on whether the voice is active at any given moment. An example may be found in mm. 35–39 (fig. 20), where the rhythmic paradigm alternates accordingly as speech–metre–speech–metre. In the manuscript, Partch delimits changes in rhythmic paradigm exclusively through the absence or presence of note stems. To increase clarity and uniformity, my transcription signals changes in paradigm with additional dashed barlines.⁶⁵ In m. 37, the Adapted Viola follows the voice exactly, changing pitch on every syllable leading up to the word “bloodily”, while the Chromelodeon moves at a slower pace, holding chords across multiple syllables. Its shift on the word “bloodily” not only doubles the voice’s pitches but also reinforces the clause’s natural emphasis and phrasing. The abutting instrumental interlude (mm. 38–39) shifts to a metred rhythmic paradigm with a half note pulse, which is notated in traditional rhythmic notation.

⁶⁴ Partch, *Genesis of a Music*, 258.

⁶⁵ Barlines drawn by Partch in the manuscript are indicated by normal solid barlines in my transcription. In general, measures in *Dark Brother* are rather long and somewhat arbitrarily defined, so regrouping the music in a consistent way based on the underlying rhythmic paradigm is useful for analysis.

Figure 20: Alternating rhythmic paradigms in mm. 35–39.
Above: Partch's manuscript; Below: HEJI transcription of the same passage.

Notably, *Dark Brother* contains no melismatic vocal passages. Each syllable of the text maps to a single tone and its natural length is not modified (e.g. through excessive lengthening) or even hinted at notationally. This is one way in which Partch ensures that the flow of words approaches a natural speech rhythm, eliminating unnecessary dwelling on any single syllable or vowel, which was one of his chief criticisms of Classical singing. Arguably, however, *Dark Brother* makes little progress in Partch's other specified fields of exploration. The metred content of the composition is generally limited to simple binary rhythms ranging from whole notes to sixteenth notes, with occasional thirty-second notes appearing as part of double-dotted rhythms. Aside from the duodecuplets comprising the Chromelodeon's figurations in part 2 of the B-section (which, anyway, are used as instrumental effect rather than rhythmic structure), no tuplets of any kind feature meaningfully in *Dark Brother*,⁶⁶ let alone rhythms of 5, 7, 11, and 13.

To understand this incongruity, one may potentially consider (at least) three explanations.

⁶⁶ In the manuscript, Partch includes triplets in the Chromelodeon's left hand in mm. 83, 85, and 88 (of the transcription, p. 8 of autograph). However, I argue that these triplets are essentially meaningless, as they will necessarily be overridden by the underlying speech rhythm paradigm.

1. We must remain open to the possibility that Partch was simply not interested in exploring complex metred rhythm in *Dark Brother*.
2. The basic binary rhythms could be a compositional device in *Dark Brother* intentionally functioning as a foil to the inherently complex rhythm of the text-driven passages.
3. The absence of such rhythms of 5, 7, 11, and 13 could be due to practical circumstances.

Unfortunately, no anecdotal evidence specifically about rhythm seems to exist to either confirm or contradict explanation 1.

Explanation 2 has some conceptual elegance, though the use of simple rhythms in conjunction with complex speech patterns is in no way exclusive to *Dark Brother*. Indeed, basic binary rhythms—as might be found in folk music or traditional children’s songs—pervade Partch’s entire compositional output from *Seventeen Lyrics by Li Po* to *The Dreamer that Remains*.

Explanation 3 may have the strongest foothold. The short section titled *Rhythmic Notation in Genesis of a Music* differs between the first and second editions in that the second contains an additional paragraph about the relationship between rhythm and percussion instruments. Partch writes, “With the introduction of percussion instruments of a rather large variety, the evolution of complex rhythms becomes almost automatic.”⁶⁷ At the time of *Dark Brother*’s composition, Partch had yet to build any of his percussion instruments. Perhaps the degree of his exploration in “the more complex forms of 5 and 7, [and] the simple impulses of 11 and 13” grew organically alongside the imminent genesis and development of his extensive collection of home-made percussion instruments (cf. works like *Delusion of the Fury* and *Daphne of the Dunes* from the 1960s).

Tonality Flux in a 43-tone 11-limit just intonation framework

Central to Partch’s logic of harmony is the concept of a *Tonality Diamond*, which he is often credited with inventing though it is based on a model initially conceived by Max F. Meyer.⁶⁸

The Tonality Diamond is a conceptual diagram that unifies the *symmetry of intervals* around a pitch of reference (1/1). Namely, intervals may be projected above or below this reference pitch, and, for the most part, this generates new pitches (or potential pitch-classes). For example, the interval of a major third 5/4 with respect to ♮G generates the pitch ♮B above, whereas the same interval inverted (4/5) produces the pitch ♭E below. Including the perfect fifth above (3/2) and below (2/3) ♮G adds the pitches ♮D above and ♮C below respectively,

⁶⁷ Partch, *Genesis of a Music*, 258.

⁶⁸ See especially Cris Forster, “The Partch Hoax Doctrines,” 2015, 11, <https://chrysalis-foundation.org/wp-content/uploads/ThePartchHoaxDoctrines.pdf>.

resulting in two segments of pitches (ratios)—a so-called “Otonality” (ascending, 4:5:6) and a so-called “Utonality” (descending, $1/4 : 1/5 : 1/6$). In musical terms, this symmetrical operation hinging on the note $\natural G$ generates the G major and C minor triads and is already well known from the perspective of Riemannian theory. If these pitches are arranged graphically such that the otonal intervals grow diagonally in the NE direction from $\natural G$ ($1/1$) and the utonal intervals grow orthogonally in the NW direction, a v-shaped structure begins to emerge (fig. 21).

$$\begin{array}{ccc}
 \frac{2}{3} (\natural C) & & \frac{3}{2} (\natural D) \\
 \frac{4}{5} (\flat E) & & \frac{5}{4} (\natural B) \\
 \frac{1}{1} (\natural G) & &
 \end{array}$$

Figure 21: Beginning structure of the 5-limit Tonality Diamond.

Similarly, adding these same otonal intervals (a $5/4$ major third and $3/2$ perfect fifth) above each of the two new pitches generated in the NE utonal segment fills in a complete “diamond” of pitches in the 5-limit (fig. 22). Note how this simultaneously fills in the corresponding utonal intervals below the pitches of the NW otonal segment—clearly, the inverse operation will produce the same Tonality Diamond.

If all of the fractions along the *same* NE diagonal are made to have a common denominator (as in fig. 23), then the Tonality Diamond’s structure is revealed more clearly. The three otonal segments along NE diagonals (4:5:6 in the numerators) produce the G, $\flat E$, and C major triads, while the three utonal segments along NW diagonals (4:5:6 in the denominators) produce the C, $\natural E$, and G minor triads. Normalising the ratios of these pitches places them all within the octave above $\natural G$ —this is the form of the Tonality Diamond that Partch preferred (because of octave equivalence), as it defines the notes of a potential Harmonic Space (i.e. scale). Ratios along the central vertical “spine” of the Tonality Diamond all correspond to $\natural G$, so the 5-limit Tonality Diamond generates a Harmonic Space with 7 unique notes. Note, however, the large gap on either side of $\natural G$.

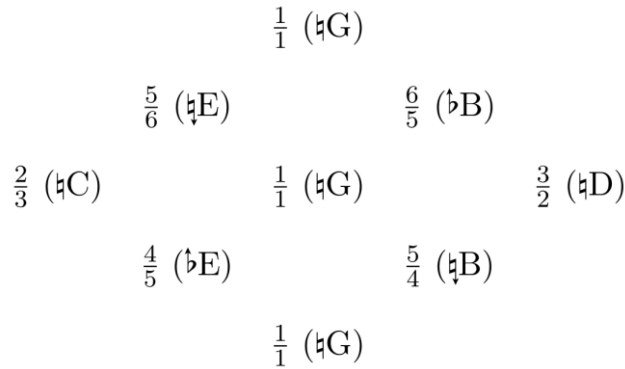


Figure 22: The 5-limit Tonality Diamond (all fractions reduced).

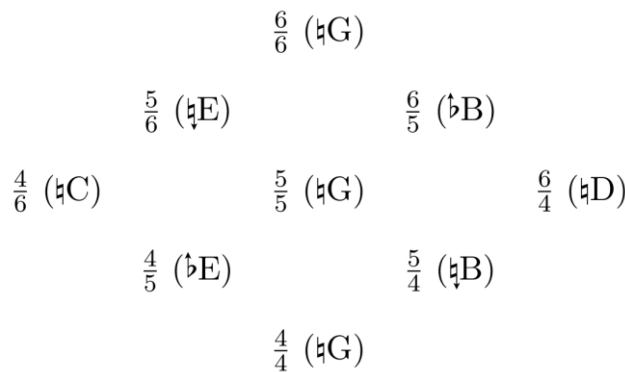


Figure 23: The 5-limit Tonality Diamond with common denominators along NE diagonals and common numerators along NW diagonals.

Since notes corresponding to even partials of the harmonic series merely equate to octave doublings of numerically smaller odd partials (e.g. $6^\circ \rightarrow 3^\circ$ in its second octave of the series), *new* notes are only be generated from by the odd partials. Partch calls these odd partials *Identities*—more specifically, he terms Identities within an Otonality *Oidentities* and those within a Utonality *Uidentities*. The 5-limit Tonality Diamond described above is constructed of U- and Oidentities 1, 3, and 5. However, for musical purposes, Partch was more concerned with the concept of intervals and ratios of frequencies than with the harmonic series (see sec. 2.2). He conceived of chords in their “orthodox”, closed-position form of stacked thirds (i.e. a “1-5-3” triad). For use in his own work, Partch expanded the Tonality Diamond to include Identities 1 through 11, constructing 1-5-3-7-9-11 hexads (see fig. 24). This 11-limit Tonality Diamond generates six Otonalities and six Utonalities, which Partch associated with the traditional major-minor duality because of their “maximum consonance for a stipulated number of

different identities.”⁶⁹ In normalised form, the twelve Primary Tonalities define a Harmonic Space with 29 unique notes. Partch refers to these Tonalities by the ratio of their “unity” or 1-Identity (i.e. ratio along the bottom-most diagonal) and whether the hexad is otonal or utonal. For example, Partch refers to the Otonality whose 1-Identity is rA as 8/7-Otonality and the Utonality whose 1-Identity is hD as 3/2-Utonality. However, in this essay, I will refer to the Tonalities by their unity pitch in HEJI notation, which I think is less abstract (and the ratios can easily be retraced).

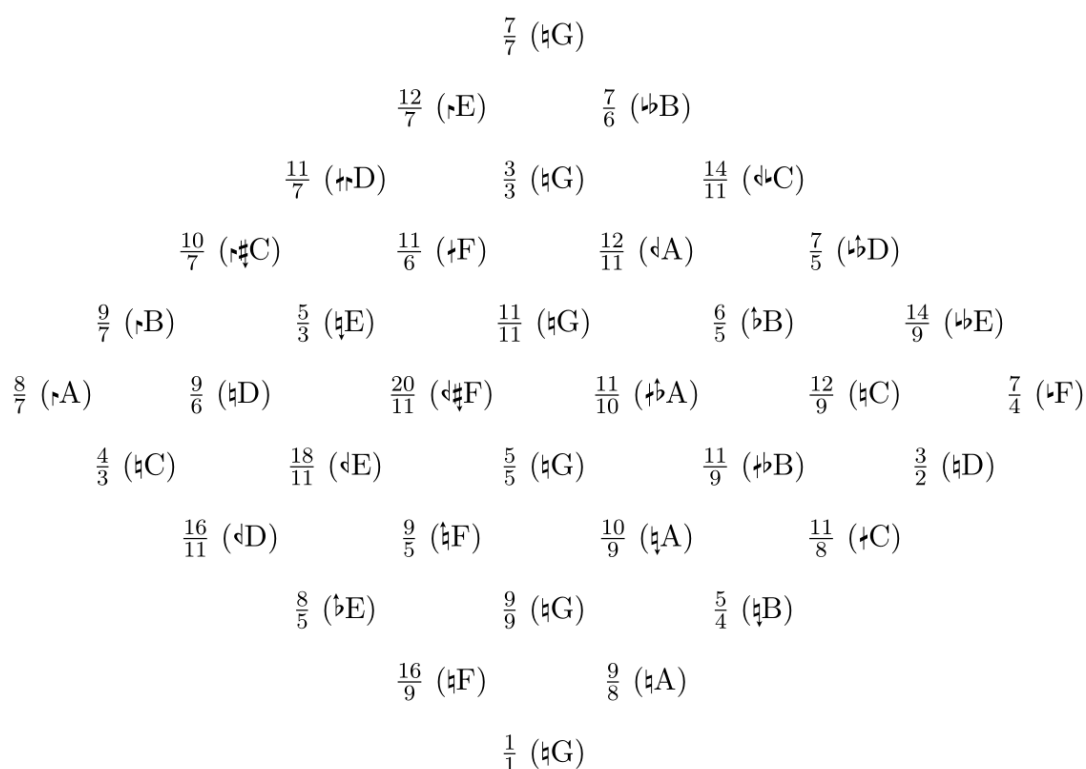


Figure 24: The 11-limit Tonality Diamond as presented by Partch in Chapter 10 of *Genesis of a Music*. Note that hC and hD both appear twice.

To create a more even distribution of tones defined by the 11-limit Tonality Diamond, Partch filled in the “larger” gaps with 14 additional intervals. These were generated by iterating small intervallic differences that were already present in the diamond, such as the syntonic comma 80:81, in less densely populated regions. The result set of pitches is a symmetrical Harmonic Space with 43 unique notes, sometimes called the *Partch Scale*. In general, the steps between adjacent notes are very fine, ranging from 14 cents (120:121) between dA and rA (or d#F and rF)

⁶⁹ Partch, *Genesis of a Music*, 160.

to 39 cents (44:45) between $\flat A$ and $\natural A$ (or $\sharp F$ and $\natural F$). These new pitches create 16 new “Secondary” Tonalities that extend outside the Diamond—most of which are not complete hexads of all six odd-number Identities like the Primary Tonalities but include subsets.

Partch’s 43-note “scale” provided him with a rich framework to represent the subtle melodic variations of speech. At the same time, the 28 corresponding Primary and Secondary Tonalities suggest a variety of possible harmonisations where a given tone may be heard with multiple “tonal senses”—i.e. the role it plays *as an Identity* within a structure may be varied. Consider how $\sharp G$ is, for example, the 1-Identity in $\sharp G$ -Tonality and the 11-Identity in $\sharp C$ -Tonality, which can be clearly discerned from fig. 24). Once all 28 Tonalities of Partch’s system are factored in, a pitch may have as few as 1 possible “tonal sense” or as many as 5–12 possible “tonal senses.”⁷⁰

The concept of “Tonality Flux” is harmonic movement within the 28 Tonalities through common tones and/or small microtonal steps, producing shifts in musical perception (tonal sense). The Chromelodeon’s linear layout across a single manual aids and encourages such harmonic movement since on average interval distance on the keyboard is approximately four times greater compared to a regular 12-note tuning.⁷¹ As such, playing progressions of typical “hand-sized” chords naturally tends toward Tonality Flux.

As one of the works most concerned with pure tonal sonority, *Dark Brother* undoubtedly represents one of Partch’s most extensive and salient explorations into Tonality Flux. I will analyse a particularly representative passage, appearing in mm. 25–41, to hopefully illustrate how it works. A harmonic reduction of this passage is reproduced in fig. 25 without intermediary passing tones that do not function as an Identity within the shifting Tonalities.

In general, Tonalities typically appear as tetrads (i.e. without all possible Identities) where the highest three tones are played by the Chromelodeon and the lowest by the Adapted Viola. The voice’s pitches are always doubled by one of these instruments. I will make use of “soprano, alto, tenor, bass” terminology to refer to specific contrapuntal voices. M. 25 begins with a Flux between $\natural D$ -Tonality and $\sharp B$ -Tonality, which share the common tone $\sharp F$, appearing in the soprano. Meanwhile, the lower three voices each move by less than a sixth of a tone: the alto and bass voices, forming a perfect fifth $3/2$, move in parallel motion up 32 cents (54:55), $\natural E$ to $\sharp E$ and $\natural A$ to $\sharp A$ respectively. In contrast, the tenor voice moves down by approximately the same amount (56:55) from $\flat C$ to $\sharp B$. Fig. 26 presents this Flux in graphic format.

⁷⁰ Partch, *Genesis of a Music*, 162.

⁷¹ E.g. while the minor third $\sharp G$ – $\flat B$ requires a shift of 3 keys in equal temperament, the $\sharp G$ – $\flat B$ analogue in Partch’s Chromelodeon tuning requires a shift of 12 keys.

Figure 25: Reduced harmonic analysis of Tonality Flux in mm. 25–41. **O** and **U** indicate if the chord is otonal or utonal in construction. The 1-Identity of each Tonality is indicated by a black notehead; if the 1-Identity is not actually present in the chord it has been indicated underneath in parentheses.

$$\begin{bmatrix} \sharp F & 5o \\ \sharp E & 9o \\ \flat C & 7o \\ \flat A & 3o \end{bmatrix} \rightarrow \begin{array}{c} \text{---} \quad \text{---} \\ 54 : 55 \text{ (+32 c)} \\ 56 : 55 \text{ (-31 c)} \\ 54 : 55 \text{ (+32 c)} \end{array} \rightarrow \begin{bmatrix} \sharp F & u11 \\ \flat E & u3 \\ \flat B & u1 \\ \flat A & u9 \end{bmatrix}$$

Figure 26: Tonality Flux between $\sharp D$ -Otonality and $\flat B$ -Utonality (m. 25). Each set of brackets encloses one of the partial Tonalties (tetrads), stacked vertically in increasing pitch-height. A “u” preceding an Identity (indicated next to its corresponding HEJI notation) indicates a Uidentity; an “o” following an Identity indicates an Oidentity. The contrapuntal Tonality Flux between the two Tonalties is indicated by an intervening melodic ratio with by its size in cents.

Into m. 26, Partch continues to use parallel motion to modulate from $\flat B$ -Otonality to $\sharp D$ -Utonality. The soprano and bass (forming the interval 12/7, a fairly wide and rough sounding major sixth) each descend by the narrow semitone 21:20, creating a shift in tonal

sense of the alto and tenor, which hold the major third $\flat B-\sharp D$. Since both $\flat B$ -Otonality and $\sharp D$ -Utonality share this major third as either 1- or 5-Identities, the parallel movement in the outer voices causes a mirror-image shift in the harmony—note the same stack of intervals constructed either upward (otonal) or downward (utonal) in fig. 27.

$$\begin{bmatrix} \sharp F & 3o \\ \sharp D & 5o \\ \flat B & 1o \\ \flat A & 7o \end{bmatrix} \begin{array}{cc} 21 : 20 & (-84 \text{ c}) \\ — & — \\ — & — \\ 21 : 20 & (-84 \text{ c}) \end{array} \rightarrow \begin{bmatrix} \sharp E & u7 \\ \sharp D & u1 \\ \flat B & u5 \\ \sharp G & u3 \end{bmatrix}$$

Figure 27: Tonality Flux between $\flat B$ -Otonality and $\sharp D$ -Utonality.

The remaining Flux in m. 26 produces a downward three-quarter-tone transposition by the 12:11 of the preceding utonal $1/7 : 1/8 : 1/10 : 1/12$ chord through an intervening Otonality. Initially, the bass serves as a common tone to introduce the $\sharp D$ -Otonality “pivot” chord. Meanwhile, the soprano voice on $\sharp E$ continues its descent, first 81 cents to $\sharp E$ (22:21) then a further 70 cents to $\sharp D$ (126:121). Here, the tenor voice rises a mere 14 cents from $\sharp A$ to $\flat A$ —the smallest interval between consecutive tones in Partch’s scale with a ratio of 120:121. The complete process is summarised in fig. 28.

$$\begin{bmatrix} \sharp E & u7 \\ \sharp D & u1 \\ \flat B & u5 \\ \sharp G & u3 \end{bmatrix} \begin{array}{cc} 22 : 21 & (-81 \text{ c}) \\ 33 : 28 & (-284 \text{ c}) \\ 11 : 10 & (-165 \text{ c}) \\ — & — \end{array} \rightarrow \begin{bmatrix} \sharp E & 9o \\ \sharp C & 7o \\ \sharp A & 3o \\ \sharp G & 11o \end{bmatrix} \begin{array}{cc} 126 : 121 & (-70 \text{ c}) \\ 112 : 121 & (+134 \text{ c}) \\ 120 : 121 & (+14 \text{ c}) \\ 12 : 11 & (-151 \text{ c}) \end{array} \rightarrow \begin{bmatrix} \sharp D & u7 \\ \sharp C & u1 \\ \flat A & u5 \\ \sharp F & u3 \end{bmatrix}$$

Figure 28: Tonality Flux between $\sharp D$ -Utonality, $\sharp D$ -Otonality, and $\sharp C$ -Utonality. The net change in each voice is equal to 33:32 (c. 151 cents), resulting in a downward transposition of the starting chord.

The role of 120:121 is of particular interest. The unusual effect of being instantly “transported” to a different branch of Harmonic Space over the course of a Tonality Flux is, perhaps, most clearly perceived when a common tone is involved. In such a modulation, one can actually *feel* its different tonal senses. This singular element of “familiarity” heightens the beautiful strangeness of this *relativity*, as if the world itself were moving around a stationary observer. As $\sharp D$ -Otonality becomes $\sharp C$ -Utonality in the above passage, $\sharp A$ and $\flat A$ effectively function as a single common tone bridging them. The small adjustment required enables movement by

completely novel routes through Partch's tonal space, while at the same time taking advantage of the *continuity* inherent to a common tone modulation. Admittedly, 14 cents is an audible step in isolation, but in the active context of Partch's composition with multiple sizes of steps fluxing to many different Tonalities, 120:121 is easily masked and begins to coalesce with the listener's sense of unison. Furthermore, given the fact that this minuscule change occurs in an exchange between Adapted Viola and Chromelodeon, any slight sense of pitch shift may just as well be perceived as being caused by simple human error or by a momentary non-linearity in the *acoustic* sound.

In these initial measures of this passage, Partch mainly uses tetradic Primary Tonalities (except for $\flat B$ -Otonality) and consistently alternates between Otonalities and Utonalities. The return to this $\flat B$ -Otonality in m. 27 with the addition of a fifth note, its 9-Odentity ($\sharp C$), produces the most "rooted" effect of any of the chords heard thus far. The addition of its 9-Odentity increases the chord's harmonic series "footprint"—i.e. increases spectral fusion; "the psychological phenomenon of finality in a single tone or chord"⁷²—without introducing any new primes. The subsequent move to $\flat E$ -Otonality is achieved by quasi-parallel motion, grounded by a shared perfect fifth $\flat B$ – $\sharp F$. With 5 of its Identities, the previously established rooted spectral fusion character is carried forward, indeed well into the next measure's shift to $\sharp F$ -Otonality (see Flux diagram in fig. 29), a befitting underscore to the text's building ferventness: "and we who sold the talent, the passion..."

⁷² Partch, *Genesis of a Music*, 160.

$$\begin{array}{c}
 \begin{bmatrix} \sharp F & 3o \\ \sharp D & 5o \\ \sharp C & 9o \\ \flat B & 1o \\ \flat A & 7o \end{bmatrix} \rightarrow \begin{array}{cc} \text{---} & \text{---} \\ 15 : 16 & (+112 \text{ c}) \\ 27 : 28 & (+63 \text{ c}) \\ \text{---} & \text{---} \\ 21 : 22 & (+81 \text{ c}) \end{array} \rightarrow \begin{bmatrix} \sharp F & 9o \\ \flat E & 1o \\ \flat D & 7o \\ \flat B & 3o \\ \sharp A & 11o \end{bmatrix} \rightarrow \begin{array}{cc} 81 : 80 & (-22 \text{ c}) \\ 36 : 35 & (-49 \text{ c}) \\ \text{---} & \text{---} \\ 54 : 55 & (+32 \text{ c}) \\ 99 : 100 & (+17 \text{ c}) \end{array} \rightarrow \begin{bmatrix} \sharp F & 1o \\ \flat E & 7o \\ \flat B & 11o \\ \sharp A & 5o \end{bmatrix}
 \end{array}$$

Figure 29: Tonality Flux between $\flat B$ -Utonality, $\flat E$ -Otonality, and $\sharp F$ -Utonality (mm. 27–29).



Figure 30: Strong missing fundamental effect produced in mm. 27–29.

Notably, these chords elicit some of the strongest psychoacoustic distortions in the piece—specifically the so-called “missing fundamentals”—outlining a kind of hidden pop song bassline (I–IV–V progression, see fig. 30). Despite the commatic “readjustment” required to move to $\sharp F$ (note the shift from $\sharp F$ to $\flat F$ (81:80) in the soprano voice), the directness of this progression “archetype”, perhaps a remnant from his early years, does not suffer and the shift is hardly noticeable. Furthermore, the Adapted Viola’s change of only 17 cents from $\sharp A$ up to $\sharp A$ (99:100), the second smallest step between adjacent pitches in Partch’s system) has a similar common tone, quasi-enharmonic effect to the melodic difference 120:121 discussed above.

As the passage progresses, more “non-Tonalities” (or perhaps distorted Tonalities), apparently reflecting the text: “our hearts were corrupted, our talents wasted and our hope gone.” In fig. 31, chords A and B do not fit any of Partch’s 28 Tonalities. Chord A is clearly an attempt to establish $\sharp F$ -Utonality (note the Chromelodeon’s $\flat B$ minor triad), however the lingering $\sharp A$ in the Adapted Viola—which is extending the 9-Uidentity of the previous $\flat B$ -Utonality—causes friction against this effort, and the Utonality is not realised “cleanly” until the end of m. 37. The music in mm. 39–40 is a reference to the preceding chorale-like texture of mm. 26–27, with a progressive transposition of the tetrad $1/7 : 1/8 : 1/10 : 1/12$ underlying the harmonic movement. A markedly dark tone accompanies the final chords of the passage, as the music seems to pull inward, “in solitude and darkness.”

31

A Vla

V

un - til our hearts were cor - rupt - ed, our tal - ent was - ted

Ch I

A (A) B

Figure 31: *Corrupted Tonalities (A and B) in mm. 32–34.*

On a broader level, Partch relies on Tonality Flux as a means of linking otonal and utonal chords, of playing with the tension between the relational perception of tones as intervals and chords on the one hand, and the overwhelming pull of the harmonic series on the other. In reality, there is an unbalance: we do not perceive Utonalities with the same unambiguity and sense of amalgamation as Otonalities. In many cases, the symmetry is imperceptible, falling in the same category as undetectable dodecaphonic row transformations. Utonalities do not form a separate yet complementary perceptual Gestalt to those of Otonalities, as they make use of the same psychoacoustic mechanisms. Instead, Utonalities come across as much more complex Otonalities—relations analogous to high harmonic partials that form, at very best, a comparatively much weaker sense of unity and at worse, a muddy sound mass.

Partch was aware of this, but his intervallic convictions of harmony led him to fixate on the fact that “the numbers of the ratios and their naturally descending inclination cannot be denied.”⁷³ Without directly suggesting that the “root” of the minor triad (i.e. Utonality) is its 1-Identity (i.e. the “fifth” of the chord in traditional music theory terminology), he merely gives the unsatisfying suggestion that “as creative music goes...the composer needs no greater authority than his fancy to put the ‘root’ wherever he wants to put it.” Notwithstanding the mostly rhetorical significance of the “root”, the fact that the traditional root of Otonality (“major”) coincides with its corresponding harmonic series fundamental further confounds our perception of Utonality. Unlike Otonality, where adding more and more Identities generally reinforces its sense of rootedness, any straggling sense of rootedness in Utonality in the form of the minor triad (which is probably more of a learned phenomenon than a psychoacoustic one) is quickly “muddled” by the addition of further Uidentities. It is intriguing

⁷³ Partch, *Genesis of a Music*, 112.

to note how, in the passage from *Dark Brother*, the Utonalities do not really produce much of a distinctive effect from the distorted, “non-Tonalities”. In particular, the “missing fundamental” effect in fig. 21 comes across so strongly in part because it is bookended by two Utonalities, which elicit no such effect.

Nevertheless, Partch’s technique of Tonality Flux—especially through common tones and even such quasi-enharmonics as those described above—provides a means of both highlighting the conceptual proximities inherent in his dual system as well as clarifying the essential differences. I believe that Partch wanted to try to bring out the intervallic nature of Utonality by focussing on its *contrapuntal* connections to other structures, in particular to Otonality. In this sense, Tonality Flux functions as a kind of tool to aid the listener in carrying an intervallic mode of listening into harmonies themselves. Whether or not this endeavour is ultimately successful concerning Partch’s employment of Utonality is perhaps not so important.⁷⁴ On the other hand, what has been important—to me on a personal level—is his steadfast focussing on the intervallic nature of harmonic sound, an aspect that easily falls to the wayside, overpowered by the fusing gravity of the harmonic series. However, these do not exist independently of one another, and discovering the right balance for a given musical idea is a substantial part of composing in extended just intonation.

2.2 Extended just intonation as spectral interaction

The somewhat fickle relationship between just intonation and the physical *fact* of the harmonic series has been aesthetic fuel in the work of many composers working with rational tuning. In *Genesis of a Music*, Partch made his position quite clear: his feeling was that *intervals*, which are based on one’s psychological engagement with relations of frequencies, serve a far clearer *musical* purpose than the monolith of the harmonic series, a common wave-of-the-hand *post facto* justification for just intonation (despite its discovery thousands of years after the principles of tuning by whole number ratios). “[D]espite the similarity, a fundamental and its partials is a different concept from the identities of a [T]onality. The first is a scientific phenomenon implicit in musical sound; the second is a psychological implicit in musical ratios.”⁷⁵ Furthermore, the Otonality–Utonality symmetry at the heart of Partch’s harmonic logic places a clear emphasis on cumulative *interval structures* to highlight tonality in speech.

⁷⁴ Part of issue is Partch’s use of closed position chords, which as Utonalities quickly get muddy. More recently, M.O. Abbott has experimented with much wider voicings of utonal structures, allowing the increased distance between pitches to leave room for the characteristic sounds of the constituent intervals. These combine to create an interesting effect similar to spectral fusion, though entirely reliant on JI sonority of the intervals themselves without eliciting any sense of “missing fundamental”.

⁷⁵ Partch, *Genesis of a Music*, 89.

No subharmonic series exists on equal footing with the harmonic series in nature from which to derive a notion of Utonality.⁷⁶

However, the concept of an extended just intonation might be thought about from, perhaps, a broader angle that is more relevant for those artists who *are* interested in the harmonic series as a potential musical framework and, especially, a correlate of *timbre*. Rather than merely extending the music's conceptual prime-limit, a listener's musical focus might be extended, quite experientially, upward in the series, compelled to confront a sound's richness of timbral interaction. In several practices, this heightened attention to spectrum is often accompanied by some kind of minimisation of materials/activity in the traditional frequency band of musical tones (a few octaves on either side of the human speaking range). This "leaves room" for the listener's ear to perceive fine spectral details, which are often brought out and enhanced by a particular way of playing (in the case of acoustic instruments).

The "clouds" in La Monte Young's monumental *The Well-Tuned Piano* come to mind as an obvious example of this "bringing out" of novel spectral activity in otherwise familiar sounds. Young builds up massive resonances through fast improvisatory articulations of static harmony (e.g. Section 3 "The Magic Opening Chord", 1:02:29–1:23:54 in the 1981 Gramavision recording), sometimes consisting of only a few pitches, other times encompassing larger chordal structures. Kyle Gann has noted the close connection with Young's Dream Houses and other sine-tone installations.

Like the installations, Young's clouds elicit impressive aural illusions. Once a cloud is set in motion, the ear may hear what sound like foghorns, voices, bells, even machinery, and often the "missing fundamental" resulting from the complexes of rationally tuned periodicities. As one moves around the room, the audible overtones change markedly over the distance of a few inches, dependent on where one is among the nodes of pitches reinforced by the acoustics of the room.⁷⁷

As a cloud gradually builds up its spectral momentum, inevitably evolving in time, various melodies of partials resulting from mixing spectra of multiple fundamentals (the "notes" Young plays on the piano) begin to take shape.

⁷⁶ The subharmonic series emerges for the most part "indirectly" in certain circumstances. One clear case results from dividing a string in equal parts: consider the collection of all node positions for a given mode of vibration of a string (the "natural harmonic" playing technique). For example, the seventh partial may be isolated at six locations (i.e. divisions) along the string: $1/7$, $2/7$, $3/7$, $4/7$, $5/7$, and $6/7$. If the player were to take note of these locations and fully stop the string instead of playing a natural harmonic at each one, the resulting sequence of stopped pitches would be a melody outlining the "subharmonic series" based on the seventh partial itself, descending downward until its sixth "undertone": $7/1$, $7/2$, $7/3$, $7/4$, $7/5$, $7/6$. For more information, see Thomas Nicholson and Marc Sabat, "Farey Sequences Map Playable Nodes on a String," *TEMPO* 74, no. 291 (January 2020): 86–97, <https://doi.org/10.1017/S0040298219001001>.

⁷⁷ Kyle Gann, "La Monte Young's the Well-Tuned Piano," *Perspectives of New Music* 31, no. 1 (1993): 149.

Éliane Radigue's acoustic works also exemplify this principle with somewhat more unusual instrumental techniques. For instance, *OCCAM IV* (2012/2017) for solo viola makes use of sustained perpendicular as well as *longitudinal* movement of the bow along the strings, "letting natural harmonics and bow-harmonics emerge."⁷⁸ In particular, the so-called "bow-harmonics" markedly equalise and filter the viola's spectrum: as the bow, traversing the string longitudinally, passes a node for a given partial, that partial and by extension its own harmonic series are emphasised in the sound while others are suppressed. Violist Julia Eckhardt, who worked with Radigue over the composition's development, writes that bowing on nodes "alters the sound in an unobtrusive manner and makes it iridescent, as if looking at it from different angles."⁷⁹ Bowing on a node of the third partial will highlight the "partials of 3", i.e. its multiples: 3°, 6°, 9°, 12°, 15°, etc. Likewise, bowing on a node of the fifth partial will highlight "partials of 5": 5°, 10°, 15°, 20°, 25°, etc.

The filtering effect of "bow-harmonics" plays a central role in the work of Ellen Fullman, who since the 1980s has been developing and investigating the unique properties of her "Long String Instrument". Consisting of multiple thin bronze or steel strings 20 metres or more in length attached to wooden resonators, Fullman brings the Long String Instrument into vibration by gently stroking the strings with rosined fingers, walking up and down its length multiple times over the course of a performance (ca. 1 metre every 7 seconds).⁸⁰ Despite the impressive length of the strings, their fundamental frequencies are not as deep as one might expect, sitting for the most part in the range of the cello. Because the strings resonate longitudinally, fundamental frequencies are surprisingly not dependent on the string's tensions but rather their material properties and their vibrating lengths, which Fullman fine-tunes with capos. A major advantage of such extreme string lengths is the increased distance between harmonic nodes. On traditional string instruments, like the viola, nodes fall within a very compressed length of string; a deviation from a node's position by even "a fraction of a millimetre will bring about a huge change or even a break."⁸¹ There is no such lack for space on Fullman's Long String Instrument.

In the summer of 2018, I had the opportunity to become much more intimately acquainted with the Long String Instrument as Fullman gave a private demonstration and even instructed me to play it myself, which was an intensely *physical* experience. While the distance between

⁷⁸ Julia Eckhardt, "On Occam IV by Eliane Radigue," 2016, https://www.academia.edu/36579151/On_Occam_IV_by_Eliane_Radigue_pdf.

⁷⁹ Eckhardt, "On Occam IV by Eliane Radigue."

⁸⁰ Ellen Fullman, "Sound Kitchen Recipes," *Ellen Fullman*, accessed January 14, 2021, <https://www.ellenfullman.com/new-page>.

⁸¹ Eckhardt, "On Occam IV by Eliane Radigue."

nodes for 5° and 4° on a viola amounts to less than two centimetres, the distance between the same nodes on Fullman's Long String Instrument is as much as a metre (or more)! Clearly, the surface area of the player's point of contact with the string (the fingertip, bow) remains the same as it would be on any other string instrument. Because of this, the Long String Instrument opens the door to an immensely expanded field of nodes for much higher partials, otherwise inaccessible on any other string instrument, that might be brought out for their timbral and melodic qualities.

Fullman also showed a highly detailed spectrogram of her instrument, revealing its astonishing spectrum of exactly harmonic partials well into the thousands. This in conjunction with the practical advantage of the instrument's length enables the production of incredibly unique auditory illusions and phenomena. The instrument's sound is "so rich in loud partials that it can sound like an ensemble of instruments, and sound like many more notes played at once than are being played. There is a lot to hear because partials sometimes even overtake the loudness of the fundamental."⁸² In her composing and collaborative music-making practice with the Long String Instrument, she uses its highly unique characteristics to "find resonances—textural miracles that are confusing as to structure but spinning and mesmerizing."⁸³

Spinning and mesmerising resonances—partials that overtake their fundamental—these are also central threads running through the work of Catherine Lamb, who brings the harmonic series into ever greater degrees of tangibility by lowering the fundamental frequency into the sub-audio range below 20 Hz. Hypothetical spectral interactions controlled by various rules and restrictions then become the actual tonal material for a composition. These are typically articulated by acoustic instruments in extended musical time, where simple forms emerge through continuity and discontinuity in the sustained sonority. Writing about Lamb's *prisma interius* series (2018–2019),⁸⁴ flautist Rebecca Lane elegantly describes this creative decision as a transposing of spectral detail "down into human dimensions."⁸⁵ Harmonically structured on either a 15 Hz or 10 Hz fundamental frequency, the *prisma interius* works centre around the *secondary rainbow synthesiser*, an instrument created in collaboration with Bryan Eubanks. The secondary rainbow synthesiser "tunes" live input signal from the immediate world outside the concert room by means of SuperCollider resonant band filters to various

⁸² Fullman, "Sound Kitchen Recipes."

⁸³ Kerry O'Brien, "A Composer and Her (Very) Long String Instrument," *The New York Times*, May 2020, <https://www.nytimes.com/2020/05/01/arts/music/ellen-fullman.html>.

⁸⁴ The *prisma interius* series comprises nine parts composed for various instrumental settings, ranging from solo to mixed large ensemble.

⁸⁵ Rebecca Lane, "CATHERINE LAMB: ATMOSPHERES TRANSPARENT/OPAQUE (Liner Notes)" (New World Records 80806-2, 2019), 9, <https://nwr-site-liner-notes.s3.amazonaws.com/80806.pdf>.

sequences of high partials of the harmonic series that it can then play as a kind of scale, bridging the two spaces both conceptually and practically. Clearly taking the earlier American experimental tradition as a springboard, Lamb's approach to this "bridging" actually goes beyond a mere affirmation of ambient noise: she searches for structures to mediate and experiment with its relationship to acoustic instruments. "[M]y use of the filtering synthesizer is to extend the filter from our innermost point and draw a connection into the outermost point, to find a thread between it all."⁸⁶

In experiencing these works, one is struck by the nuanced, deep listening that the "outermost point" invites. Lane writes, "At its narrowest point, the filter produces the effect of concentrating the musical object [a single frequency] inside the walls of the space...As the filter opens, our ears extend outward and begin to identify sounds (construction works, children's voices, car engines)."⁸⁷ But aside from eliciting their recognition, these "noises" initiate a further level of somewhat evanescent sub-filtering in the processed sound. Like bowing on a node, these clear aberrations in the exterior sound field create moment-to-moment boosts in energy at certain frequencies, which translate into subtle emphasising and/or attenuating of particular branches of related partials in its timbre. One experiences continually transforming harmonic perspectives of the sonic whole (the extended harmonic series), like shifting shadow and light.

⁸⁶ Che Chen, "KRAAK: Catherine Lamb," 2019, <https://kraak.net/avant-guardian/catherine-lamb>.

⁸⁷ Lane, "CATHERINE LAMB," 4–5.

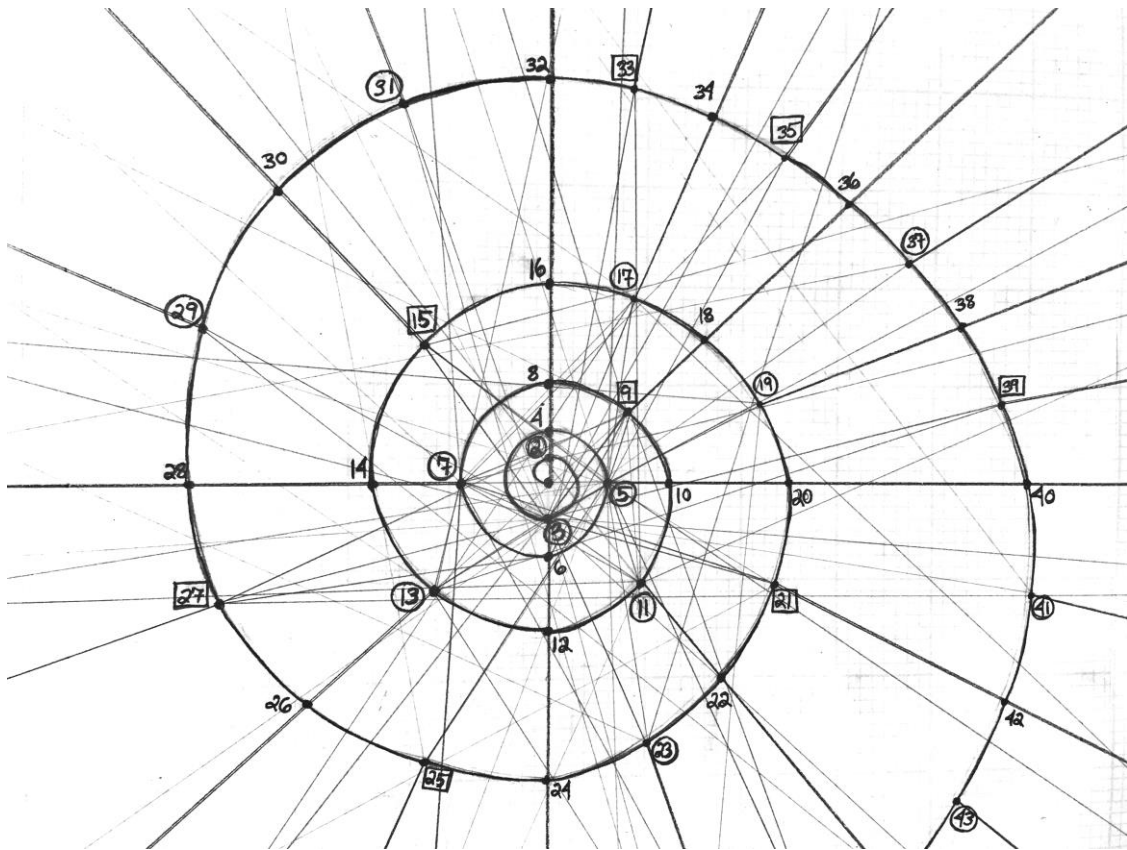


Figure 32: One of Lamb's many spiral diagrams.

While openness to uncertainty is clearly called for by the secondary rainbow synthesiser in the *prisma interius* pieces,⁸⁸ Lamb has found means of creating analogous, complementary kaleidoscopic tonal shiftings for a far more concrete domain of acoustic instruments, composing speculative counterpoints of constrained spectral interactions, extended in time, that often arise from working with her “spiral”. Expanding from a graphic visualisation published by Erv Wilson in 1965,⁸⁹ Lamb has developed a compositional and pedagogical practice around a spiral model of the harmonic series where each octave maps onto progressive “cycles” of an expanding logarithmic spiral (fig. 32). “It [Wilson’s spiral] was congruent with my own early visualizations of numbering. Numbers were large scale points in

⁸⁸ “Working with the unpredictability of what might occur I find quite fascinating and makes it less about myself within the chaos and more myself in direct interaction with the chaos by filtering it a certain way.” Chen, “KRAAK.”

⁸⁹ Erv Wilson, “The Harmonic Series as a Logarithmic Spiral,” 1965, <http://www.anaphoria.com/harm-subharm.pdf>.

space shooting upwards in (at first) a seemingly straight line. As the numbers increased, curves began to form and return and return in cyclical mannerisms.”⁹⁰

Lamb’s string quartet *divisio spiralis* (2019) is one of many composition projects resulting from her working with and contemplation of the spiral. Reminiscent of the chance-based filtering of the secondary rainbow synthesiser—circumstantial boosting/attenuating of closely related partial chains—Lamb “play[s]...with the nodal branching/simplified series and the more modal relationships [suggested by the spiral]...I [Lamb] guess this is a bigger question: the higher the primes next to one another, the more modal it feels?”⁹¹ Lamb’s tonal framework supports this hypothesis. The historical modes, when conceived as single strings of partials, are only found in higher regions of the series. For example, the basic 5-limit major scale only first occurs as 24:27:30:32:36:40:45:48—and more complex modes with more complex tunings will necessarily lie in much higher regions. As Lane pointed out, the clear advantage of bringing the fundamental deep into the sub-audio domain, 10 Hz in the case of *divisio spiralis*, is that such modal relationships that only live high in the series, whether due to high primes and/or unusual chains of intervals, become much more audible. In other words, they are able to take advantage of *harmonic auditory cognition*, a term coined by Marcus Pal to describe our ability to perceive and parse the finely nuanced and characteristic sound of just intonation intervals (melodic and harmonic), which is not necessarily restricted to, but is considerably augmented in the mid-band (ca. 60–1000 Hz). Below, I offer a rough analysis of the *divisio spiralis*’s first four sections (pp. 1–27) as an illustration of some of the points discussed above.

⁹⁰ Catherine Lamb, “Conceptualizing the Spiral: Humans Intersecting with Harmonic Space (an Introduction Towards a New Pedagogical Pathway),” *Živá Hudba* 11 (2021). Preprocessing (publication in late 2021).

⁹¹ Catherine Lamb, personal communication (January 17, 2021).

2.2.1 Catherine Lamb's *divisio spiralis* (2019)

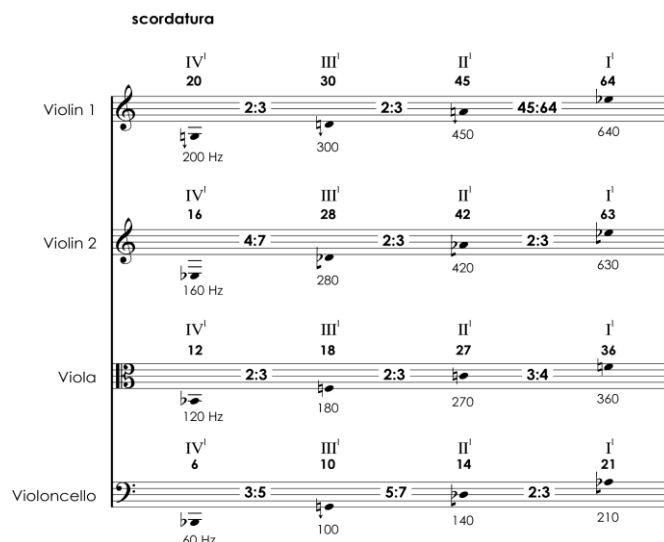


Figure 33: The tuning of the quartet's open strings in Lamb's *divisio spiralis*.

Concerning the 10 Hz fundamental, Lamb imposes a “palindromic” structure of restrictions by which its partials are filtered: i.e. may or may not feature as tones in the composition, articulated by the string quartet. In this way, she sets up a kind of “basic timbre” or conceptual “ambient noise” of the work, which is altered in various ways over the course of the work. The instruments' open strings are retuned to various partials in the series (ranging from 6°–64°, see fig. 33), enabling them to access the partials by either playing stopped notes or natural harmonics. Broadly speaking, though *divisio spiralis* is in 29-limit just intonation (involving all primes ≤ 29), not all multiples of the relevant primes appear in the work. In other words, Lamb's 10 Hz series is not complete in the 29-limit. Instead, an additional layer of prime-limits restricts how prime numbers *multiply with each other*. This generates filtered and reduced branches of Harmonic Space: partials of prime 29 are restricted to the “1-limit” (i.e. to 29° itself); partials of 23 are restricted to the 2-limit (i.e. to octave transpositions, 23°, 46°, etc.); those of 19 are restricted to the 3-limit (i.e. to the Pythagorean multiples); and so on, with lower primes having the most extensive trees. This framework is summarised in tbl. 3. Clearly, not all of the partials in a given tree are unique to it: those shared by multiple trees might serve as useful connections in a prime-limit transition. For example, partial 105°, which is $3 \times 5 \times 7$, appears in each of the trees generated by those three primes.

Tree generator	Prime-limit of its multiples	Example partials in tree
29	“1-limit”	29° only
23	2-limit	23°, 43°, 92°, etc.
19	3-limit	19°, 57°, 152°, etc.
17	5-limit	17°, 85°, 102°, etc.
13	7-limit	13°, 65°, 91°, etc.
11	11-limit	11°, 77°, 121°, etc.
7	13-limit	7°, 49°, 77°, 105° , etc.
5	17-limit	5°, 55°, 105° , 125°, etc.
3	19-limit	3°, 81°, 105° , 189°, etc.
2	23-limit	2°, 64°, 112°, etc.

Table 3: Palindromic structure restricting Harmonic Space in Lamb’s *divisio spiralis*. The partial 105° in bold illustrates one of many connecting partials shared between multiple trees.

On a metaphorical level, the minimalistic global plan of *divisio spiralis* seems to emulate the widening frequency band of the secondary rainbow synthesiser. Beginning in the concentrated region surrounding 96° (960 Hz), the quartet’s harmonic material and range of frequencies expand little by little. Progressively lower partials are introduced, culminating with the cello’s lowest string (60 Hz, i.e. 6°). The expansive pace of the music gradually slows even further as the piece progresses and “as harmonicity expands”⁹² One has the impression of listening to some sound object that has been processed with various band filters, focussing on particular regions and sub-structures of its spectrum. In this sense, *divisio spiralis* might be seen as something of a (re)continuation of James Tenney’s *Arbor Vitae* (2006), also for string quartet, which—although in a different way—is also concerned with branching harmonic structures in the series of a single fundamental and has a similar textural roadmap.⁹³ Whether intentional or merely a correlation, this association is nevertheless a nice link to Tenney, who was an important teacher and mentor to Lamb in the early 2000s when she studied at CalArts.

Divisio spiralis opens in section 1 with a heterophonic investigation of the partials immediately around 96° (fig. 34). All four instruments follow and react to one another, imitating each other’s melodic ideas in close temporal proximity (relatively speaking) or reinforcing certain

⁹² Catherine Lamb, program note to *divisio spiralis*.

⁹³ *Arbor Vitae* begins very high in its series and gradually descends into ever lower partials before finally reascending to the upper-register. For a detailed analysis, see Michael Winter, “On James Tenney’s *Arbor Vitae* for String Quartet,” *Contemporary Music Review* 27, no. 1 (February 2008): 131–50, <https://doi.org/10.1080/07494460701671566>.

frequencies by momentarily extending them—freezing them in time, highlighting them in the 10 Hz series. In combination with the concentrated, high register (around 960 Hz), the many microtonal overlappings and shearings, which are constantly evolving, create an effect not unlike shimmering light. Gradually, the bandwidth begins to open and more extended melodic threads emerge, a formal process used by Lamb in earlier works (cf. the voice's extended opening sequence in *parallaxis forma* (2016) for voice and ensemble). At the same time, the harmonic complexity, or perhaps *saturation* of the opening pages (23-limit—recall that prime 29 only occurs once much lower in the series at 29° itself) is increasingly reduced as the higher primes are progressively filtered out. Eventually, the music is limited to only partials in the 7-limit by page 6 and finally the 5-limit by the end of the section (pp. 9–10, see fig. 35). A fleeting appearance of a single partial of 11 in the cello (110°) closes the section, which evaporates into the first moment of silence in the piece.

divisio spiralis
Catherine Lamb

①
(sounding one octave higher than written)

Violin I: 96 92 96 98 91 98 96 91 90 Π^2 90
 Ratios: 23:24, 14:13 vla 1:1 vc, 13:14 vll 1:1 vc, 1:1 vll 1:1 vla, 15:16 vla 9:10 vc, 1:1 vll

Violin II: 96 98 91 96 , 96 90 100
 Ratios: 1:1 vl, 1:1 vl/vc, 1:1 vl, 1:1 vl, 16:15 vl, 1:1 vl, 1:1 vc 10:9 vl

Viola: 96 92 96 91 96 98 91 96
 Ratios: 1:1 vl, 23:24, 48:49, 1:1 vll, 1:1 vll

Violoncello: Π^7 98 , 100
 Ratios: 14:13 vla, 25:24 vla

Figure 34: The opening of *divisio spiralis*. A practical advantage of Lamb's 10 Hz fundamental is that the frequency of each partial p is simply $10 \times p$. Thus, she finds it useful to include partial number indications in her scores (written above each pitch), which simultaneously serve an analytic/conceptual function as well as a purely practical function for any musicians using tuners for reference. This not only de-clutters the page but also avoids the need for cent indications in her scores, freeing her music from any references to equal temperament. It is worth point out that Lamb uses her own "style" of HEJI notation that involves far more ligatures of symbols. Their relation to the standard implementation of HEJI (appendix B) is nevertheless easily discerned. Note, for instance, how she attaches the septimal symbols to the flat to notate 98° (in standard HEJI this would appear as $\flat C$).

8

V I 108 4:3 vc 9:7 vla

IV⁴ 80 1:1 vll

III³ IV⁴ 90 80 9:8 1:1 vll

III³ 90

V II (96) , 96 108 96 84 80 84 80 90 100 96 100 90 80 96 90

8:9:7 vl/vla 1:1 vl 8:9 vl 1:1 vla 5:6 vc 1:1 vla 5:6 vc 9:8 vl 15:16 vla 10:9:8 vl 1:1 vla 10:9:8 vl 1:1 vl 1:1 vl 1:1 vla 1:1 vl

Vla 84 7:8 vll

IV⁸ 96 1:1 vc

Vc (81) 84 96 1:1 vla 1:1 vll

III⁸ 80 1:1 vl/vll

9

(90) IV⁵ 100 1:1 vll 5:4 vc

IV⁴ 80 1:1 vc

V II 100 96 , 80 90 100 110 100

5:4 vc 1:1 vla 1:1 vc 1:1 vla 5:4 vl/vc 11:10:9:8 1:1 vc 5:4 vl

Vla IV⁸ III⁵ 96 90 16:15 9:8 vc

Vc (80) III¹¹ 110 1:1 vll

10

Figure 35: The closing pages of section 1.

The following sections 2–4 (pp. 11–27) begin a sequence of contrapuntal threads of filtered partials articulated by moments of silence that lasts for the remainder of the composition. Once again, as if bowing on nodes of a monumental string tuned to 10 Hz, Lamb seems to seek out the modes and melodic contours inherent to her system when certain resonant bands are “boosted”—in other words, when specific prime “trees” are isolated in conjunction with a constant underlying Pythagorean backbone (primes 2 and 3). For instance, p. 11 has a clear focus on primes 5 and 7, which expands to include 13 (meanwhile 5 is momentarily filtered out) on p. 12. The harmony then shifts to a fully saturated 11-limit space by p. 13. It is interesting to acknowledge the importance of the unit of “the page” and its direct influence as a structural delimiter for the music’s harmonic “colour” at any given moment: a symbiosis between musical structure and musical practicality (the JACK quartet, for whom the piece was composed, plays with iPads!). The page almost has the role of an impartial mediator of *when* harmonic change occurs. Fig. 36 summarises these prime focuses page by page of the score.

section	page	prime focus					
		5	7	11	13	17	19
2	11	■	■				
	12		■		■		
	13	■	■	■			
	14		■				■
	15		■		■		
	16	■					
3	17	■					
	18	■	■				
	19	■				■	■
	20	■	■	■			
4	21	■	■		■		
	22	■	■				
	23	■	■				
	24	■	■				
	25	■	■	■			
	26	■	■	■	■		
	27	■	■	■	■		

Figure 36: The sequence of specific prime focuses in sections 2–4 of *divisio spiralis*. Primes 29 and 23 do not appear in this passage.

For the most part, the unfolding of the work’s harmonic transformations through various branches of the spiral is approached through intuition:

In a way I [Lamb] worked pretty intuitively down along the spiral...every new tone arriving in the descent initiates a new shift to the color, but then also simply moving through the palette I had set up, whatever seemed interesting in a moment...⁹⁴

However, as the work progresses, the ear's tonal sensitivity adjusts to the larger-scale microtonal details, ascertaining certain structures that closely link filtered regions come into the musical foreground, which in some ways resemble Tonality Flux. I will provide two illustrations.

On p. 20, the first violin comfortably establishes 121° ($\sharp\sharp\flat D$), which momentarily lingers into the p. 21 as prime 11 is filtered out and the harmony expands to include prime 13 (fig. 37). Here, Lamb plays with a potential ambiguity—clearly, as one moves higher in the series, the perceived interval between neighbouring partials becomes smaller, *approaching a unison*. Though 121° is a partial of 11 (namely 11×11) and should have technically been filtered out as the shift in focus moved from that prime, its proximity to partial 120° ($\sharp D$) less than a twelfth of a tone lower creates a strong perceptual thread linking the two pitches. This ambiguity is clarified only after the harmonic shift has taken place, as the first violin finally adjusts to 120° through a subtle portamento. In a sense, 121° and 120° function as quasi common tones ($\sharp\sharp\flat D \approx \sharp D$) in a similar manner to how Partch utilised the very same frequency ratio in moments of Tonality Flux in *Dark Brother*. Meanwhile, the shift in harmonic context is solidified by the second violin, which iterates its melodic contour from the previous page mapped to the new “mode”.

⁹⁴ Catherine Lamb, personal communication.

④

The musical score consists of four staves: V I, V II, Vla, and Vc. The notation includes various musical symbols such as notes, rests, and ratios. The ratios are as follows:

- V I:** 66 (1:1 vc), 77 (7:6 vll/vc), 88 (4:3 vc), 99 (3:2 vc), 110 (1:1 vc), 121 (11:8 vll).
- V II:** 66 (1:1 vl), 77 (1:1 vl), 88 (4:3 vc), 99 (9:7 vla), 110 (1:1 vc), 121 (1:1 vl), 99 (9:7 vla), 88 (8:7 vla), 77 (7:8 vla), 66 (3:4 vla).
- Vla:** 77 (1:1 vll, 7:6 vc), 88 (4:3 vc), 77 (7:11), 88 (1:1 vll).
- Vc:** 66 (1:1 vl), 110 (5:3), 66 (1:1 vl), 110 (5:3).

20

The score continues with the following ratios:

- V I:** (121) 24:13 vll, 20:13 vla, 112 (16:13 vla), 117 (3:2 vll).
- V II:** 65 (5:6 vla), 78 (6:7 vla, 6:5 vc), 91 (7:6 vc), 104 (4:3 vc), 117 (1:1 vl), 104 (4:3 vla), 96 (16:13 vla), 104 (4:3 vla), 117 (1:1 vl), 104 (4:3 vla, 8:5 vc).
- Vla:** (88) 78 (6:5 vll), 91 (7:5 vll/vc), 84 (14:13 vll/vc), 78 (1:1 vc, 2:3 vl).
- Vc:** 65 (1:1 vll), 78 (1:1 vll, 2:3 vl), 91 (7:6 vla), 78 (1:1 vla), 72 (3:4 vll), 64 (2:3 vll), 65 (5:6 vla).

Figure 37: Neighbouring partials 121° and 120° (violin 1) create a sense of ambiguity as harmonic focus moves away from prime 11 to prime 13.

Lamb highlights another wonderful enharmonic-like ambiguity between partials 125° ($\sharp D$) and 126° ($\flat E$) in the transition from p. 22 to p. 23, shown in fig. 38. In this case, 125° is clearly established, this time by the second violin, within the 5-limit “mode” of p. 22. However, the listener’s perception of the decisively 5-limit character of 125° —emphasised by the underlying $\flat B$ major tonality on p. 22—gradually comes into question with the appearance of 72° ($\natural F$) in the first violin’s counterpoint on p. 23. The ear fights to simplify the relationship between 72° and 125° , which pulls toward the familiar harmonic seventh with the ratio $7/4$, hearing 125° as a potential mistuning. Lamb, who appears to be completely aware of this perceptual conflict, allows the second violin to adjust its pitch by just 14 cents up to 126° (note that $72:126 = 4:7$). The harmonic transformation to the 7-limit creeps in, unfolding with a sense of inevitability.

These “bridges” of neighbouring partials— $121:120$ and $125:126$ —are actually the *subtlest* differentiations in pitch encountered in the whole composition.⁹⁵ And yet, they prove to play a decisive role in bringing about particular shifts in harmonic perspective and colour. In my own work, I have found that approaching a kind of Tonality Flux based on interval relations from the perspective of interacting partials high in the harmonic series (potentially much higher than those highlighted above) has led me to certain structures that have reactivated that initial sense of wonder I felt when I experienced just intonation for the first time. In the next chapter, I will describe some of these structures and provide examples from my recent compositions where I develop musical frameworks to realise them.

⁹⁵ The most distant point in the series reached during *divisio spiralis* is partial 153° (first violin on p. 54). Due to the restraints imposed by Lamb’s limiting of the way primes multiply (tbl. 3), there are only two pairs of neighbouring partials remaining whose perceived difference is *smaller* than those highlighted above (namely $135:136$ and $152:153$); neither of these possible “bridges” are used by Lamb.

Figure 38 is a musical score for a string quartet, consisting of four staves: V I, V II, Vla, and Vc. The score is divided into two systems, with the first system ending at measure 22 and the second system starting at measure 23.

Staff V I: Features a treble clef and a key signature of one flat. It includes a III^4 marking at the beginning and a III^8 marking at the end. Ratios and instrument abbreviations are provided below the staff: 24:13 vll, 15:13 vc, 5:3 vla, 5:4 vc, 5:4 vll.

Staff V II: Features a treble clef and a key signature of one flat. It includes a III^{10} marking at the beginning and a III^8 marking at the end. Ratios and instrument abbreviations are provided below the staff: 5:6 vl, 1:1 vc, 3:4 vl, 1:1 vc, 24:13 vll, 15:13 vc, 6:5 vla, 8:5 vla, 1:1 vl, 8:5 vla, 24:25 vc, 3:4 vl, 4:3 vla, 1:1 vl, 3:2 vla.

Staff Vla: Features a treble clef and a key signature of one flat. It includes a III^{10} marking at the beginning and a III^8 marking at the end. Ratios and instrument abbreviations are provided below the staff: 3:4 vll, 2:3 vl, 4:5 vc, 3:4 vl, 4:5 vl/ll, 3:4 vll, 2:3 vll, 4:5 vc, 3:4 vc, 2:3 vl, 4:5 vc, 5:6 vll.

Staff Vc: Features a bass clef and a key signature of one flat. It includes a III^{10} marking at the beginning and a III^8 marking at the end. Ratios and instrument abbreviations are provided below the staff: 1:1 vll, 1:1 vla.

Staff V I (continued): Features a treble clef and a key signature of one flat. It includes a I^1 marking at the beginning and a I^2 marking at the end. Ratios and instrument abbreviations are provided below the staff: 4:3 vla, 6:5 vla, 1:1 vc, 1:1 vla, 4:5 vc, 1:1 vla, 1:1 vc, 1:1 vla, 3:2 vla, 4:3 vll.

Staff V II (continued): Features a treble clef and a key signature of one flat. It includes a I^2 marking at the beginning and a I^3 marking at the end. Ratios and instrument abbreviations are provided below the staff: 3:2 vc, 4:3 vc, 3:2 vla, 4:3 vla, 1:1 vc.

Staff Vla (continued): Features a treble clef and a key signature of one flat. It includes a I^2 marking at the beginning and a I^3 marking at the end. Ratios and instrument abbreviations are provided below the staff: 1:1 vl, 1:1 vl, 1:1 vl.

Staff Vc (continued): Features a bass clef and a key signature of one flat. It includes a I^2 marking at the beginning and a I^3 marking at the end. Ratios and instrument abbreviations are provided below the staff: 21:20, 2:3 vll, 2:3 vll, 1:1 vla, 8:9 vla, 1:1 vla, 1:1 vla, 1:1 vll, 4:3 vla, 1:1 vll.

Performance instructions are provided below the staves: "spectral (less fundamental) light bow" and "fundal tone normal bow".

Figure 38: Neighbouring partials 125° and 126° (violin 2) facilitate the smooth emergence of p. 23's focus on prime 7 out of the previous page's 5-limit framework.

Chapter 3: A More Adaptive Extended Just Intonation

3.1 Enharmonic proximity: Tonality Flux on the smallest scale

The widely held assumption that any music for acoustic instruments composed in extended just intonation is inevitably limited to ratios fixed to a certain “key” or fundamental—beyond which any significant explorations and modulations cause “undesirable side-effects” like so-called *comma drift*—has the potential to lead a composer to a creative dead end. If a piece modulates to ever more distant regions of Harmonic Space, a composer may begin to feel somewhat conceptually inhibited, left searching for fresh compositional means of moving the music forward. Does one restrict the music to only a handful of simple harmonic moves? Or does one allow the tonal centre to “drift as needed”—Lou Harrison’s *Free Style* approach—at the risk of possibly losing track of the connection between perception, physicality, and notation, or forcing it to become excessively complex and unmanageable? In such circumstances, it might appear that just intonation as a practical tone system *exclusively* offers a kind of modulatory linearity whereby, unless one chooses to simply move yet deeper into Harmonic Space, the only alternative is to “rewind the tape” and follow the same (or a very similar) harmonic logic backwards.

In my own music, a remedy has frequently been to allow for extremely slight loose ends, which can lead to musically fruitful and typically unexpected harmonic experiences. Approximately four years ago, I began to reflect more carefully on this dilemma in conjunction with Partch’s principle of Tonality Flux. More specifically, I wondered if—in the context of a more nuanced and varied just intonation Harmonic Space than Partch’s 43 tones—some of the small steps between tones involved in a Tonality Flux might be reducible to almost nothing, in the same way that the distance between neighbouring partials approaches unison the higher one looks in the harmonic series. Such near-unisons could conceivably take advantage of the *enharmonic* properties of temperament without the need to falsify either harmonic or melodic

intervals. This had the potential to become a novel technique of creating harmonic shifts, thereby offering a “way out” in otherwise boxed-in situations through a harmonic treatment that is usually considered “unsupported” by a traditional fixed just intonation.

I refer to *enharmonic proximities*—a term coined by Marc Sabat for points in Harmonic Space that are composed of vastly different prime factors but nearly equivalent pitch-heights.⁹⁶ In other words, enharmonic proximities are complementary pitches with nearly equivalent pitch-distances (perceived interval sizes) from a tone of reference, but different harmonic lineages. A small melodic step separates them that reminds of their different relations to the larger Harmonic Space; this is often *tempered out* in the process of creating musical temperaments. There is, however, nothing dictating that these differences *must* be tempered out, as contexts may arise that facilitate their perception as well as their playability. “This happens,” as Sabat has written, “because the musical point of reference is not always perceived as [M]onophonic, in the sense of Harry Partch (a fabric derived from and always respecting one originating pitch), but rather as a shifting sequence of reference points, each related to one another.”⁹⁷

A common method of handling these situations is *adaptive just intonation*, which retains the exact just ratios of simultaneous sounds, but uses tempered melodic intervals to reduce the retuning motion and/or drift that arises from modulation. While this approach may be effective for a certain type of predominantly diatonic and triadic music in which a more conventional 5-limit just intonation might be appropriate, it is hard to justify the need for (or benefit of) melodic tempering in the context of a new music that seeks a rich exploration of harmonic nuance and its innate properties. Instead, the use of *untempered* enharmonic proximities where close connections are needed or suggested allows the music to “swing” the short distances between branches of Harmonic Space, without “covering up” or “evening out” the various idiosyncrasies inherent to extended just intonation. The result is neither a Free Style approach nor (irrationally) adaptive just intonation, but a kind of hybrid of the two: the tonal reference is allowed to drift until it may be *reset* or *reinterpreted* through a rational, enharmonic proximity, thereby resolving (or at least alleviating) many notation and practical difficulties that may arise in a complex rational tone system. As a composer who is heavily influenced by notation as both a conceptual and practical tool, I find such instantaneous

⁹⁶ In this context, the term “enharmonic” assumes the relatively contemporary meaning relating to two notes that are “interchangeable”. For example, in 12-tone equal temperament, sharps are *exactly* enharmonic with flats on the next diatonic whole tone (e.g. $\sharp G = \flat A$). In just intonation, no two tones are exactly enharmonic (i.e. equivalent) in the strict sense (aside from the trivial case of two tones with the same ratio).

⁹⁷ Marc Sabat, “THREE TABLES FOR BOB,” *Tempo* 70, no. 278 (October 2016): 53, <https://doi.org/10.1017/S0040298216000334>.

“simplifications” and “reinterpretations” a great source of creative inspiration, suggesting a multiplicity of directions the harmony might follow—seemingly at my fingertips.

As for small melodic gaps that separate nearly coinciding pitches in Harmonic Space, a search for potential enharmonic proximities might begin with the syntonic comma. Consider a very simple example: the pitches $10/9$ and $9/8$ are guaranteed according to the fundamental theorem of arithmetic to have different prime factorisations ($2^1 \times 3^{-2} \times 5^1$ and $2^{-3} \times 3^2$ respectively) and, therefore, occupy different regions of Harmonic Space. However, they have similar pitch-heights measured from $1/1$ (182 cents and 204 cents respectively), differing by the syntonic comma 80:81 (22 cents). In composition, this proximity might suggest possible bi-directional, enharmonic-like substitutions, occasionally shifting aspects of the harmony by an eighth-tone. These could propel a piece into variously transforming harmonic directions if *not tempered out*, but rather allowed to flourish as an integral structural characteristic.

However, at nearly an eighth-tone, the syntonic comma is, admittedly, a rather “large” step. In most circumstances, it is clearly perceived as separating *different notes* that may somehow function as an interchangeable pair within a specific musical context—more in line with Partch’s original applications of Tonality Flux. In recent time, I have been interested in enharmonic proximities that are separated by steps much smaller than the syntonic comma, blurring the perceptual boundary between them, becoming something like enharmonic “equivalencies”. An early example of this type of tonal exchange is in James Tenney’s *Harmonium #1* (1976) in which the 17th and 24th partials of an $\sharp F$ fundamental swap roles to become the 12th and 17th partials of a $\sharp B$ fundamental (fig. 39). Because Tenney assumes *tempered* fundamentals, this is achieved through an almost negligible adjustment of 3 cents in each voice (in the top two staves). In a pure just intonation context, however, where the fundamentals would form a Pythagorean diminished fifth $\sharp F-\sharp B$ (1024/729), the adjustment is actually a bit larger at 8.7 cents (2176:2187) but still extremely subtle.

In my own composing, I have found that the small step 224:225 (8 cents) is something of a baseline: on the very edge of perception, this difference is both the largest gap between pitches that might still come across as an equivalency while also being harmonically useful (one of the primary relations bridging the 5- and the 7-limits). Consider a simple example. In terms of a harmonic reference ($1/1$) defined as e.g. the note $\sharp D$, the major seventh ($15/8$) above the major third ($5/4$) may effectively function as a kind of equivalency to the harmonic seventh ($7/4$) above the perfect fourth ($4/3$). See fig. 40 for this example in notation.

Figure 39: HEJI transcription by Marc Sabat of the enharmonic shift (over the first double barline imaged) in James Tenney's *Harmonium* #1.

Figure 40: A possible construction of 224:225, in this case descending (225:224).

Technically, of course, the difference between $\sharp E$ and $\flat F$ is not negligible, but limitations imposed by acoustic instruments—certainly in many practical and musical contexts—make this differentiation rather ambiguous. Bearing this in mind, such a minuscule gap has great musical potential in composition, offering a rich palette of re-imagined harmonic logic that actually straddles the border between temperament and extremely precise just intonation. One may initially treat proximities as “tempered equivalencies” in some sense while allowing for fine melodic adjustments that would otherwise be tempered out to take place at the last second *as they become tuneable*—i.e. perceivable. This might entail fine adjustments through the elimination of beating, the stabilisation of the waveform (i.e. searching for periodicity), the emphasis of the common partials, etc. As these differences are so fine, they necessitate a very decisive musical articulation to become perceptible. Notating such differentiations offers an interpretative opening for “expressive” treatment on a project to project basis: either the


difference is “made clear”, or it is deliberately “blurred” or “shaped” through, for instance, a small pitch-bend.

Figure 41: Enharmonic “bridge” 225:224 in *BRANCH* (Plainsound Trio).

Such expressive treatment may often be made more practical with the addition of a third voice, acting as a kind of linchpin. In mm. 43–44 of my piece *BRANCH* (Plainsound Trio) (2018) for any three instruments (fig. 41), 225:224 is set up in the same manner as described previously. Instrument 3’s melodic semitone 15:16 from $\sharp F$ to $\natural G$ is accompanied by a middle voice ($\natural A$) to produce the dyads 6/5 and 9/8. Meanwhile, instrument 1’s enharmonic step down a third-comma from $\sharp E$ (tuned above $\sharp F$) to $\flat F$ (*adjusted* to the 7th partial of the series of $\natural G$) gradually *comes into focus* as instrument 2 ascends a fourth from $\natural A$ to $\natural D$. In other words, both chords, whether with $\sharp E$ $\flat F$, have similar probabilities of exhibiting a *particular* psychoacoustic effect, namely the simple triad 4:6:7.⁹⁸ The result is a fleeting moment of ambiguity or “fuzziness” at the beginning of m. 44 that progressively “sharpens” to form a crisp (and in some sense *inevitable*) septimal “image”. Like a physical force, this technique sets harmonic experience *in motion*. It is neither confined to a static, unyielding just intonation nor acquiescing to the tonal approximations of any given temperament. The act of fine-tuning is, itself, acknowledged as an active element of the musical language, rather than superficially hidden behind the scenes.

Enharmonic proximities play a particularly structural role in my piece *GRAM* (2018) for Baroque alto recorder, viola, and keyboard (in Pythagorean tuning). The initial seed for the

⁹⁸ This relates to the concept of *Harmonic Entropy*, a probabilistic measure of perceived concordance developed by Paul Erlich that, in his own words, analyses the question, “how confused is my brain when it hears an interval?” See Paul Erlich, “Harmonic Entropy,” *Sound of India*, 2004, accessed February 14, 2021, http://www.soundofindia.com/showarticle.asp?in_article_id=1905806937 and William A. Sethares, “Appendix J: Harmonic Entropy,” in *Tuning, Timbre, Spectrum, Scale*, 2nd ed. (Berlin: Springer Verlag, 2004), 355–357.

piece was the observation that a stack of eleven descending septimal commas ($64/64$)¹¹ (–299.9 cents) almost exactly divides the descending tempered minor third (–300 cents). After some initial experiments trying to formulate a piece around this descending stack, I decided to take an enharmonic route instead. Rather than dividing the *tempered* minor third, I desired to keep the piece clear of any references to temperament by dividing the nearby Pythagorean minor third $27/32$ (–294 cents). Given the notational challenges imposed by writing up to eleven septimal commas (which would look like “”), I decided to re-map the exact ratios of descending “scale” of septimal commas to various 13-limit enharmonic proximities that were both easier to notate and more comprehensible on a practical level—a kind of “rational temperament”. The result was a symmetrical “reverse *KOAN*”, detailed in tbl. 4. Naturally, this completely transformed the piece’s trajectory: rather than using these pitches to attempt to *represent* the scale of septimal commas in my initial concept, which I anyway found more creatively restricting, I followed the unique character of each new enharmonic proximity and its relationship to the piece’s global 1/1 (the viola’s D-string) to *lead* the harmony on a more varied journey through Harmonic Space.

Septimal commas (63/64)				Enharmonic proximities			
<i>Exponent</i>	<i>Ratio</i>	<i>Cents</i>	<i>HEJI</i>	<i>Ratio</i>	<i>Cents</i>	<i>HEJI</i>	<i>Error (c)</i>
0	$\frac{1}{1}$	0	♮D	$\frac{1}{1}$	0	♮D	±0
1	$\frac{63}{64}$	−27.3	♭D	$\frac{63}{64}$	−27.3	♭D	±0
2	$\frac{3969}{4096}$	−54.5	♭♭D	$\frac{32}{33}$	−53.3	♭♭D	+1.3
3	$\frac{250047}{262144}$	−81.8	♭♭♭D	$\frac{243}{256}$	−90.2	♯C	−8.4
4	$\frac{15752961}{16777261}$	−109.1	♭♭♭♭D	$\frac{15}{16}$	−111.7	♯♯C	−2.7
5	$\frac{992436543}{1073741824}$	−136.3	♭♭♭♭♭D	$\frac{12}{13}$	−138.6	♯♯C	−2.3
6	$\frac{62523502209}{68719476736}$	−163.6	♭♭♭♭♭♭D	$\frac{117}{128}$	−155.6	♭♯C	+8.0
7	$\frac{3938980639167}{4398046511104}$	−190.9	♭♭♭♭♭♭♭D	$\frac{9}{10}$	−182.4	♮C	+8.5
8	$\frac{248155780267521}{281474976710656}$	−218.1	♭♭♭♭♭♭♭♭D	$\frac{8}{9}$	−203.9	♮C	+14.2
9	0.867851021982492	−245.4	♭♭♭♭♭♭♭♭♭D	$\frac{891}{1024}$	−240.9	♯B	+4.5
10	0.854290849764016	−272.6	♭♭♭♭♭♭♭♭♭♭D	$\frac{6}{7}$	−266.9	♯B	+5.8
11	0.840942555236453	−299.9	♭♭♭♭♭♭♭♭♭♭♭D	$\frac{27}{32}$	−294.1	♮B	+5.8

Table 4: “Scale” of eleven septimal commas re-mapped to a symmetrical set of 13-limit enharmonic proximities in GRAM (2018). Because of the very large numbers beyond 8 septimal commas $(63/64)^8$, ratios are given in decimal form. Aside from 8/9, all of the proximities are within ca. a third-comma of the exact septimal ratios.

In mm. 75–76 of movement IV “Harmonium for Ben Johnston” from Marc Sabat’s string quartet *Euler Lattice Scenery Spirals* (2011–12), the 5-limit Euler lattice⁹⁹ is folded in upon itself by means of the enharmonic connection 10460353203/10485760000 (4 cents) between e.g. ♮E and ♭F (fig. 42). This “seam”, as Sabat refers to it in his performance notes, makes it possible for a return to the diatonic Pythagorean notes by the end of the movement along a completely *different path* in the lattice than was used in the first half of the movement. Not unlike the classic arcade game *Asteroids*, a trajectory to an edge region of the Euler lattice leads directly to a *re-entry* (through an extremely fine microtonal adjustment) from the opposite edge—a flat projection of a torus topology.

⁹⁹ The Euler lattice (*Tonnetz*) is a two-dimensional graphic representation of 5-limit triadic pitch-class space. Typically, points along the *x*-axis (the “3 dimension”) indicate projections of 3/2 perfect fifths and points along the *y*-axis (the “5 dimension”) indicate projections of 5/4 major thirds.

Un poco largo
♩ = 57.6

(dim.) +4.2 c -35 libero p ppp < " > change bow as needed -49 -51 poco *f* e sempre sostenuto +4.2 c +4.2 c -49 -51 (dim.) sotto voce (diminuendo on F only)

Figure 42: Marc Sabat connects distant regions of the Euler lattice through proximities 4 cents apart.

What if proximities are so near in pitch-height space that the melodic steps separating them are effectively imperceptible? For me, this is an interesting question because it extends the concept toward a true kind of “perceptual equivalency” that effectively requires no physical action to realise. In *BRANCH*, I also work with enharmonic proximities separated by the micro-step 728:729 (2 cents), only 1% of a tone. In the passage from mm. 22–27 (fig. 43), this enharmonic relationship links very distant regions of 13-limit Harmonic Space, almost as if the music were to travel through a wormhole. The effect is akin to the surprise Jean-Philippe Rameau describes when hearing the impressive modulatory acrobatics enabled by the diminished seventh chord, which quickly “turns into admiration after finding oneself thus transported from one hemisphere to the other...without having had the time to think about it.”¹⁰⁰

¹⁰⁰ “...cette surprise se tourne en admiration, de se voir ainsi transporté d’un Hémisphere [sic] à l’autre, pour ainsi dire, sans qu’on ait eu le tems [sic] d’y penser.” Jean-Philippe Rameau, *Génération Harmonique* (Paris: Pault fils, 1737), 153.

Figure 43 shows a musical score for three staves (1, 2, 3) across two systems. The first system (measures 21-25) features a 'meno forte' dynamic. The second system (measures 26-27) includes a '26 : 27 = +65c' marking. The score uses various musical notations, including notes, rests, and pitch-bending symbols (arrows and numbers).

Figure 43: Enharmonic step 728/729 in BRANCH (Plainsound Trio).

Examining the modulation with the help of a kind of reduced pitch-height diagram provides a useful visualisation (fig. 44). In grey, the underlying baseline is shown as the clear $\flat C$ fundamental initially established in m. 22. After modulating to the closely related $\flat B$ fundamental (partial 7 $^\circ$ of $\flat C$) over the next measures, the modulation to arrive at the distant fundamental $\sharp D$ is shown in black. This unlikely jump to such a complex ratio in few steps (729/640) is quite easily achieved through what is in essence a common-tone modulation requiring a small theoretical adjustment of 2 cents by instrument 1 (m. 26). The note $\flat G$ —the 13th partial (13 $^\circ$) of the $\flat B$ 7/4 fundamental—transforms into Pythagorean $\sharp F$ —the 5th partial (5 $^\circ$) of the $\sharp D$ 729/640 fundamental. Alternatively, since the middle voice facilitates the modulation from below the enharmonic substitution by tuning to the instruments around it, a utonal perspective is also possible, where Pythagorean $\sharp F$ becomes the “utonal fundamental” (common partial) of $\sharp D$, its “under 5” (u5). From either angle, the unusual modulation is achieved through essentially no physical action on the part of the performer. I previously

wrote “theoretical adjustment” because is merely taken for granted that the 2 cent difference is within the realm of the instrument’s own physical instabilities (bowing, breathing, slight inharmonicity, etc.) and that instrument 2 may tune $\sharp D$ as if instrument 1 had indeed been playing $\sharp F$ all along.

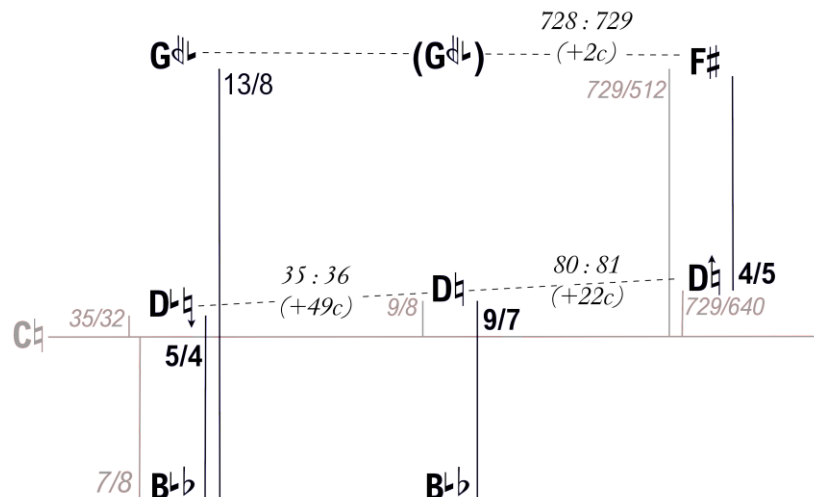


Figure 44: Pitch-height diagram of changing harmonies in BRANCH (Plainsound Trio).

The properties of such near “foldings” of Harmonic Space upon itself are conventionally used as cues for introducing temperament to the *entire* region, begging the question: Should an instrument capable of this degree of precision be implemented with a specific temperament? An example of this conceptual distinction may be found historically in the difference between Hermann von Helmholtz’s *schismatic* micro-temperament and Arthur von Oettingen’s temperament built on a similar principle for his Orthotonophonium (an example of which exists in the Musikinstrumentenmuseum Berlin). The schisma (32768:32805) is the small difference between eight untempered perfect fifths (octave-reduced) and a pure major third—amounting to ca. 2 cents. Helmholtz’s tuning achieves pure major thirds ($5/4$) by reducing *each* of eight perfect fifths by an eighth of this schisma (just a quarter of a cent) while Oettingen’s allows for seven untempered perfect fifths followed by one perfect fifth tempered by an *entire* schisma to achieve pure major thirds. Clearly, in a comparison where the maximum difference is only 2 cents, the distinction is truly subtle. However, it articulates the paradox (or, even, false dichotomy) inherent to harmonic experience of *any* tone system, whether just or tempered. Should the tonal centre remain fixed or should it be allowed to slightly drift (as “untethered” instruments unavoidably do)? Should the resulting tone system then be deliberately “irrationalised” or, alternately, artificially “rationalised”? Perhaps exclusively subscribing to either “system” or denying connections suggested by perception does not really conform to actual auditory cognition, which is malleable and whose modalities

(sense and reason) are deeply and mutually dependent, as Claudius Ptolemy so thoughtfully reminded us two millennia ago in his passage about *harmonia* (referenced in ch. 1): “it is in general characteristic of the senses to discover what is approximate and to adopt from elsewhere what is accurate, and of reason to adopt from elsewhere what is approximate, and to discover what is accurate.”

Beyond the techniques of enharmonic *adjacency* I have described up to this point, an enharmonic proximity may just as well appear *harmonically* as a (fixed) pitch being used to represent another pitch (outside the strict tuning) with a small error, i.e. *without* fine adjustments. In my quartet *ANSTATT* (2019), commissioned by the Paul Hindemith Musikschule in Berlin as a glimpse into extended just intonation for young music students, I momentarily extend Pythagorean space into the enharmonic 11-limit. Throughout most of this composition for clarinet, lightly retuned accordion (to produce untempered perfect fifths), violin, and cello, the accordion remains within the realm of the Pythagorean diatonic “white notes” $\sharp F$ through $\sharp B$, generated by a chain of fifths. In m. 8, however, it moves through a further fifth in the Pythagorean chain— $\sharp F$ —within a harmonic field of septimally lowered flats in the other instruments (fig. 45). Here, the interval between the cello’s $\flat A$ and the accordion’s $\sharp F$ (1047 cents) is less than 3 cents shy of a pure neutral sixth $11/6$ (1049 cents). In combination with all of the instruments, $\sharp F$ creates a striking passing 11-limit “purr” (effectively producing 6:9:11:12) within an otherwise septimally-oriented passage. The error of 3 cents is somewhat perceivable, causing a slight, even pleasant momentary periodic beating, though there is no mistaking the “true” tone it is meant to stand in for ($\sharp F \rightarrow \flat\flat G$).

This effect was inspired by the (fortuitously named) *Toccata settima* (1657) by Michelangelo Rossi, which exploits a remarkable (and rarely explored) trait of quarter-comma meantone temperament. Some augmented/diminished intervals produce excellent approximations of septimal chords, with a typical error of 2–3 cents (fig. 46). Assuming a quarter-comma meantone tuning logically centred on D (as described in ch. 1, fig. 6), all of the augmented 6th chords in the example indicated with arrows sound unambiguously like 4:5:7. Notably, the septimal chords in mm. 65 and 66 “resolve” upward, foreshadowing Tartini’s suggestion of the same (also discussed in ch. 1).

3.2 Search for enharmonic proximities in 23-limit Harmonic Space

Both fascinated and inspired by the harmonic possibilities suggested by such extremely near enharmonic proximities, I began a more systematic search to build a collection of melodic steps that could facilitate potentially interesting and compositionally useful modulations. In Sabat's article *Three Tables for Bob*, which includes a shortlist of 19 such enharmonic connections, he writes: "some pitches or ratios that may easily be heard and tuned in some circumstances end up requiring a visual notation that is excessively laden with signs."¹⁰¹ Since the symbols required to notate a given enharmonic shift may be divided *between* the tones involved, enharmonic proximities often provide a helpful tool for actually simplifying such notational difficulties (see the discussions of *BRANCH* and *GRAM* above). After all, the ear may often much more readily *hear* relationships than the brain may *parse* a string of signs, *convert* it to an auditory expectation, then attempt to *test for* or *produce* that expectation. As such, I limited my search to enharmonic proximities in 23-limit Harmonic Space requiring no more than seven HEJI alteration symbols shared between the two tones. In my experience, intervals and chords within the 13-limit are generally the most salient types of harmonic sounds, producing well-defined psychoacoustic effects, while some extensions through 17, 19, and 23 may also be tuned with good accuracy depending on the timbres used, volume, the degree to which the harmonic context reproduces a simple harmonic spectrum, etc. Relations with prime factors beyond 23, however—while extremely colourful and *melodically* expressive—are for the most part not tuneable (in close positions, anyway) with the precision required for very fine harmonic shifts. Lastly, although including up to seven HEJI alteration symbols is probably pushing the limits of practical manageability, it nevertheless ensures that none of the simpler (and likely most useful) enharmonic proximities are overlooked while also suggesting some more intricate options.

I used this basic framework to create a simple Python search algorithm in which I refined the search to ratios of whole numbers ≤ 9999 , step sizes no greater than a quarter-comma (ca. 5.4 cents), and steps resulting from *epimoric ratios*, i.e. ratios with the form $a:(a+1)$. Limiting the search to steps smaller than a quarter-comma sets the largest enharmonic to the interval 324:325. And since epimoric ratios represent steps between neighbouring partials in a harmonic series, it seemed to me that they offered the maximum amount of harmonic difference while remaining small in terms of interval size, the hope being to take advantage of the greatest degree of modulatory *leverage*. Included epimorics were limited to prime

¹⁰¹ Sabat, "THREE TABLES FOR BOB," 53.

factorisations containing no more than 12 powers of 2, 8 powers of 3, 4 powers of 5, 3 powers of 7, 2 powers of each 11, 13, 17, and 19, and 1 power of 23.

This algorithm produced 87 results. I sorted the entries according to both increasing prime-limit and increasing exponential degree to create a catalogue with smoothly increasing complexity, included as appendix D. As a further measure of harmonic “accessibility”, I marked proximities with a black box if they:

- require no more than 4 HEJI alteration symbols (shared between the two tones);
- have no factors of 5, 7, 17, or 19 if *both* 11 and 13 are factors;
- do not combine 17 with 19;
- have no factors of 11, 13, 17, or 19 if 23 is a factor.

These are the most salient harmonic relations in the catalogue. A secondary collection of moderately complex, though potentially interesting, exceptions to these criteria is also suggested, indicated with grey boxes. Those entries that reproduce Sabat’s enharmonic proximities are highlighted with a bold row number.

I propose the artful use of enharmonic proximities in music composition, both melodically and harmonically, as one means of thoughtfully interacting with Ptolemy’s dictum. Indeed, most of the enharmonic proximities calculated by my algorithm have, as far as I am aware, not been explored *untempered* in music composition. A few of the entries have been useful in my pieces, having for the most part cropped up through intuition or sometimes chance.

Employing a simple algorithm to extend the more common proximities has the advantage of generating results that may otherwise, for whatever reason, elude one’s own harmonic intuition. Perhaps many of the more obscure entries will prove to be impractical in some way, though some have already begun to find musical applications—for instance, M.O. Abbott has created an intriguing modulation hinging on 3025:3024, a shift of about half a cent, in his chamber piece *Vⁱ* (2020) (fig. 47). I look at this catalogue as a large compositional project for myself and anyone else who may find it compelling—no enharmonic need be ruled out until musically put to the test. With unrestricted freedom, creativity has a way of finding uses for even the most unlikely resources.

Figure 47 is a musical score for a piece by M.O. Abbott. It features six staves: Clarinet (Cl), Trombone (Tbn), Guitar (Gtr), Violin (Vln), Viola 1 (Va 1), and Viola 2 (Va 2). The score is written in a complex, non-standard notation with various accidentals and time signatures. Key annotations include:

- Cl:** 15/4 Gtr A +31, 1260 : 1331 = +95¢, 11/5 Vln Bb +26, 242 : 75 = -2028¢ (c. 29 : 9).
- Tbn:** 165 : 66 = -1871¢ (c. 50 : 17), 8064 : 15125 = +1089¢ (c. 8 : 15), 11/5 Va1 5/6 Vln +45, 7/11 Vln -2, 9/11 Tbn.
- Gtr:** 7:06, G -43, 4/5 Gtr, 7:41, 3025 : 3024 = -0.57¢, 392 : 275 = -614¢ (c. 10 : 7), 8:10, 4/11 Vln +29, 4/7 Cl.
- Vln:** E -6, A -39, 9/4 Tbn, 9/5 Gtr, 14/9 Vln, 14/5 Gtr E +25, 7/2 Tbn, 8/5, 504 : 605 = +316¢ (c. 5 : 6), 6/11 Cl, 6/5 Vln, C -23, 14 : 15, A# -19, 9/7 Tbn, 847 : 675 = -392¢ (c. 5 : 4), 15/4 Gtr +17.
- Va 1:** 14/9 Vln, 14/5 Gtr E +25, 7/2 Tbn, 8/5, 504 : 605 = +316¢ (c. 5 : 6), 6/11 Cl, 6/5 Vln, C -23, 14 : 15, A# -19, 9/7 Tbn, 847 : 675 = -392¢ (c. 5 : 4), 15/4 Gtr +17.
- Va 2:** Ab +12, 7/11 Va1, 7/4 Gtr, 98 : 121 = +364¢.

Figure 47: In V^1 , M.O. Abbott uses 3025:3024 (in violin) to connect an extremely remote region of Harmonic Space to progressively more familiar territory (note the simplifying accidentals).
Score notated at sounding pitch.

A first attempt on my own part at a more comprehensive structural exploration of the enharmonic proximities suggested by this research has been Part 3, “CHANTER pour Wolfgang von Schweinitz”, of my work for solo cello *VIRER BERCEUR CHANTER* (2019–20), composed for French-Canadian cellist Émilie Girard-Charest. Over the course of this movement, which is a kind of extended meditation on harmonic change, eight enharmonic proximities ranging from 8 to 2 cents in size serve to fold the harmonic fabric in various unusual yet distantly familiar ways. Specifically, these proximities are (in order of appearance) 224:225, 384:385, 350:351, 624:625, 728:729, 324:325, 539:540, and 440:441. The piece is composed of a series of loops in which the contrapuntal consequences of one or more of these enharmonic proximities is/are explored. One of these steps, 224:225, is too large to have been found by my Python algorithm, but it is a characteristic, almost essential connection to include in an extended just intonation exploring such near-enharmonic relations in Harmonic Space (as discussed above). The others provide equally pivotal harmonic leverage. For example, the remarkable upward reorientation by just 5 cents (324:325) in m. 16 from $\sharp\flat B$ to $\sharp B$ allows for an instantaneous return to the Pythagorean pitches required for a seamless connection back to the open string $\sharp A$ (fig. 48). I find such shifts in harmonic perspective simultaneously surprising, at least on a conceptual level, and yet wholly familiar—perhaps inevitable—due to their perception as effectively the same pitch.

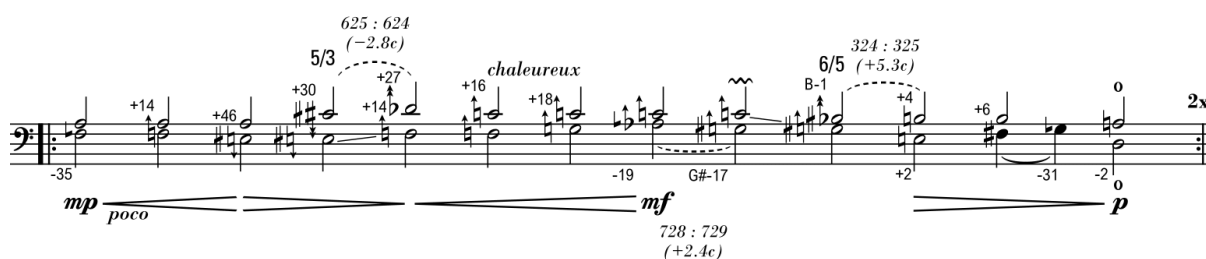


Figure 48: Enharmonic reorientation by 324:325. This loop also contains enharmonic proximities 625:624 and 728:729.

However, certain harmonic relationships must be set up pre-emptively to gain access to these enharmonic proximities. A quick glance at their prime factorisations reveals an abundance of powers of 5, 7, and 13 in particular. I allowed this fact to organically influence the types of sounds (dyads) I used when constructing progressions, and found a special overall focus on the dyads 9/7 (435 cents) and 13/10 (454 cents), which sound like unusual, fairly exaggerated (wide) major thirds. Though these sounds are complex and take practice to learn how to tune accurately, their juxtaposition with the pure major third 5/4 and perfect fourth 4/3 between which they are nested and their interplay with other intervals and pitches over the course of a phrase (especially through enharmonic shifts) clarifies and indeed solidifies their identities as distinct, tuneable sounds. A clear example may be found in m. 12 (fig. 49), where a simple, almost banal thematic idea shifts smoothly from a “5-limit flavour” to a “7-limit flavour” through the interval 13/10. This is possible because the pitch 13/10 above $\sharp F$, i.e. $\sharp B$, is only 5 cents (351:350) sharper than the septimal tuning $\flat B$. Ending the phrase with the dyad 9/7 between $\flat F$ and open $\sharp A$ allows for a momentary overlapping of the two tuning “flavours” in the melodic domain on the repeat: $\sharp A$ returns to $\sharp B$ through the classic 5-limit semitone 15:16 while $\flat F$ in the lower voice descends the narrow septimal minor third 7:6 to open $\sharp D$. The result is an unusual sounding superposition of relatively benign elements, an expressive by-product of the use of enharmonic proximities that I find particularly appealing.



Figure 49: Enharmonic reorientation by 351:350.

Generally, I take the position that any concern for the growing “number of notes” is not really so important (obviously aside from the physical constraints of some instruments—though, as

I have discussed, special options are available in even these circumstances). The music I have described is foremost guided by a deep and dynamic *listening*, which many good performers already do well and, with time and thoughtful direction, may learn to do with even greater care. It is also driven by the desire to make the most of Harmonic Space as well as our subtle abilities to perceive sound, of which, quite frankly, the majority of music until today (including much so-called microtonal music) has only begun to scratch the surface. In any event, there is usually some instrument present in any given ensemble able to produce a desired tone or facilitate a desired harmonic modulation, no matter how intricate. This leads to a unique compositional aesthetic where all aspects of the music, even orchestration, are guided by properties of Harmonic Space and a composition's movement through it. As the quantity and, more importantly, the quality of music composed in extended just intonation grows exponentially—building upon the respected though comparably small repertoire of classic works by composers like Harry Partch, Ben Johnston, La Monte Young, Lou Harrison, and James Tenney—the fact that *good music attracts good performers* (and vice versa) will continue to strengthen its foothold in all areas where music is both enjoyed and contemplated.

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Appendix A

HEJI legend and harmonic series example

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020 | LEGEND

revised by Marc Sabat and Thomas Nicholson | PLAINSOUND MUSIC EDITION | www.plainsound.org

in collaboration with Wolfgang von Schweinitz, Catherine Lamb, and M.O. Abbott, building upon the original HEJI notation devised by Marc Sabat and Wolfgang von Schweinitz in the early 2000s

PYTHAGOREAN JUST INTONATION | generated by multiplying / dividing an arbitrary reference frequency by PRIMES 2 and 3 only

... ♭♭ ♭ ♮ # × ...

notate a series of **perfect fifths** above / below a reference

$3/2 \approx \pm 702.0$ cents (i.e. 2c wider than tempered)

each new accidental represents 7 fifths, altering by one apotome

$2187/2048 \approx \pm 113.7$ cents

Frequency ratios including higher prime numbers (5–47) may be notated by adding the following distinct accidental symbols.

Custom indications for higher primes or various enharmonic substitutions may be invented as needed by simply defining further symbols representing the relevant ratio alterations.

PTOLEMAIC JUST INTONATION | PRIMES up to 5

♭♭ ♭ ♮ # × ♭♭ ♭ ♮ # ×
♭♭ ♭ ♮ # × ♭♭ ♭ ♮ # ×
~♯ = ♭ ~♭ = #

includes the consonant **just major third**

$5/4 \approx \pm 386.3$ cents (ca. 14c narrower than tempered)

alteration by one syntonic comma

$81/80 \approx \pm 21.5$ cents

alteration by two syntonic commas

$81/80 \cdot 81/80 \approx \pm 43.0$ cents

alteration by one schisma to notate an exact enharmonic substitution

$32805/32768 \approx \pm 2.0$ cents

SEPTIMAL JI | PRIME 7

♭ ♮
♭ ♮

includes the consonant **natural seventh**

$7/4 \approx \pm 968.8$ cents (ca. 31c narrower than tempered)

alteration by one septimal comma (Giuseppe Tartini)

$64/63 \approx \pm 27.3$ cents

alteration by two septimal commas

$64/63 \cdot 64/63 \approx \pm 54.5$ cents

UNDECIMAL | PRIME 11

♭ ♮

includes the **undecimal semi-augmented fourth**

$11/8 \approx \pm 551.3$ cents (ca. 51c wider than tempered)

alteration by one undecimal quartertone (Richard H. Stein)

$33/32 \approx \pm 53.3$ cents

TRIDECIMAL | PRIME 13

♭ ♮
~♭ = ♭♭ ~♯ = ♯♯
≡♭ = ♭♭ ≡♯ = ♯♯

includes the **tridecimal neutral sixth**

$13/8 \approx \pm 840.5$ cents (ca. 59c narrower than a tempered major sixth)

alteration by one tridecimal thirddtone (Gérard Grisey)

$27/26 \approx \pm 65.3$ cents

combination of $11/13$ re-notated enharmonically

alteration by the ratio $352/351 \approx \pm 4.9$ cents

combination of $11 \cdot 13$ re-notated as a single symbol

alteration by the ratio $144/143 \approx \pm 12.1$ cents

PRIMES 17 THROUGH 47

≈ ≡
↘ ↙
↓ ↑
⇓ ⇑
↙ ↘
♭ ♮
- +
▼ ▲
♭ ♮

alteration by one 17-limit schisma

$2187/2176 \approx \pm 8.7$ cents

alteration by one 19-limit schisma

$513/512 \approx \pm 3.4$ cents

alteration by one 23-limit comma (James Tenney / John Cage)

$736/729 \approx \pm 16.5$ cents

alteration by one 29-limit sixthtone

$261/256 \approx \pm 33.5$ cents

alteration by one 31-limit quartertone (Alinaghi Vaziri)

$32/31 \approx \pm 55.0$ cents

alteration by one 37-limit quartertone (Ivan Wyschnegradsky)

$37/36 \approx \pm 47.4$ cents

alteration by one 41-limit comma (Ben Johnston)

$82/81 \approx \pm 21.2$ cents

alteration by one 43-limit comma

$129/128 \approx \pm 13.5$ cents

alteration by one 47-limit quartertone

$752/729 \approx \pm 53.8$ cents

CENTS HEJI accidentals may be combined with an indication of their deviation in cents from equal temperament as read on a tuning meter; A♭ 440 Hz is usually defined to be ± 0 cents. If this deviation exceeds ± 50 cents, the nearest tempered pitch-class may be added: e.g. A♭⁻⁶⁵ (-65 cents from A♭) could include the annotation A♭⁺³⁵ placed above or below its accidental.

TEMPERED NOTES | may be combined with cents deviations to notate free microtonal pitches

... ♭♭ ♭ ♮ ♯ × ...

indicate the respective equal tempered quartertone;

show which pitch is assigned a deviation of 0c

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

*in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard orton notation above A₄

8 partial interval alteration (+2)

5° M3 (-14) (81:80)

7° m7 (-31) (64:63)

11° P4 (+4) (32:33)

13° M6 (+51) (27:26)

17° aug8 (-2) (2187:2176)

19° m3 (+4) (512:513)

23° aug4 (+51) (729:736)

29° m7 (+28) (256:261)

31° P8 (+2) (32:31)

A#-47 (-45)

37° M2 (+51) (36:37)

41° M3 (+29) (81:82)

43° P4 (+12) (128:129)

47° aug4 (+51) (729:752)

Standard utonal notation below E₄

5 M3 (+2) (80:81)

7 m7 (+16) (63:64)

11 P4 (+33) (33:32)

13 M6 (+2) (26:27)

17 aug8 (+2) (2176:2187)

19 m3 (+4) (513:512)

23 aug4 (+16) (736:729)

29 m7 (+31) (261:256)

31 P8 (+29) (31:32)

37 M2 (+2) (37:36)

41 M3 (+47) (82:81)

43 P4 (+4) (129:128)

47 aug4 (+16) (752:729)

Appendix B

Tuning charts for Partch instrument

Tuning of Harry Partch's Chromelodeon I

details from *Genesis of a Music* (1949)

transcribed into Helmholtz-Ellis JI Notation by
Thomas Nicholson

G4 is tuned to 392 Hz

(i.e. concert A4 = 441 Hz)

Chromelodeon I

sounding pitch

keyboard pitch

A^L [8']

Z [16']

(11/9) (11/9) (128/105) (27/22) (100/81) (27/22) (60/49) (49/40) (11/9) (243/200) (40/33) (77/64)

X [4']

⁶/₅ [cp.]

A^R [8']

(135/112) (40/33) (40/33) (880/729) (6/5) (6/5) (77/64) (6/5) (6/5) (40/33) (121/100) (40/33) (6/5)

(6/5) (77/64) (6/5) (6/5) (880/729) (40/33) (40/33) (132/112) (77/64) (40/33) (243/200) (11/9)

(8)

The musical score is presented in five systems. Each system consists of two staves: the top staff represents the 'sounding pitch' and the bottom staff represents the 'keyboard pitch'. The notation includes various accidentals (sharps, flats, naturals) and JI ratios (e.g., 11/9, 128/105, 27/22, 100/81, 60/49, 49/40, 243/200, 40/33, 77/64, 135/112, 121/100, 132/112, 243/200, 11/9) placed above the notes. The first system is labeled 'Chromelodeon I' and includes 'A^L [8']' and 'Z [16']'. The second system includes 'X [4']', '⁶/₅ [cp.]', and 'A^R [8']'. The third system includes a large '8' with a dashed line. The fourth system includes a large '8' with a dashed line. The fifth system includes a large '8' with a dashed line.

Tuning of Harry Partch's First Kithara

as used in his early works ("Dark Brother", etc.)

*transcribed into Helmholtz-Ellis JI Notation by
Thomas Nicholson*

G4 is tuned to 392 Hz
(i.e. concert A4 = 441 Hz)

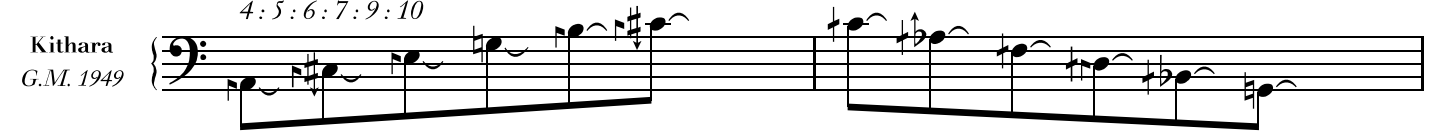
Kithara
G.M. 1949

1: 8/7-0

4: 5: 6: 7: 9: 10

2: 11/8-U

1/4: 1/5: 1/6: 1/7: 1/9: 1/11

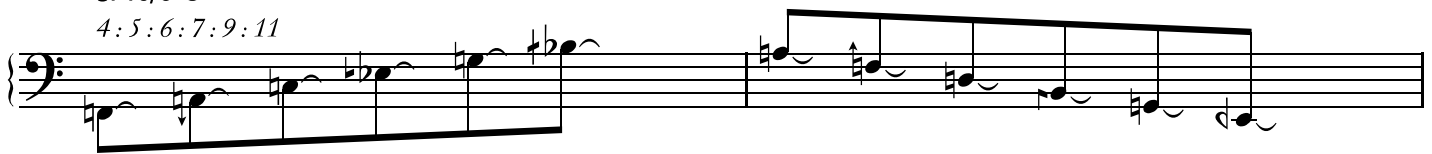


3: 16/9-0

4: 5: 6: 7: 9: 11

4: 9/8-U

1/4: 1/5: 1/6: 1/7: 1/9: 1/11

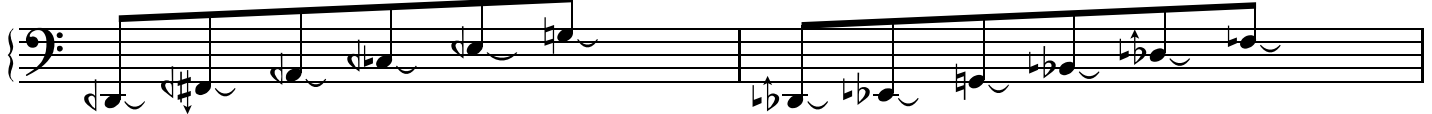


5: 16/11-0

4: 5: 6: 7: 9: 11

6: 7/4-U

1/10: 1/9: 1/7: 1/6: 1/5: 1/4

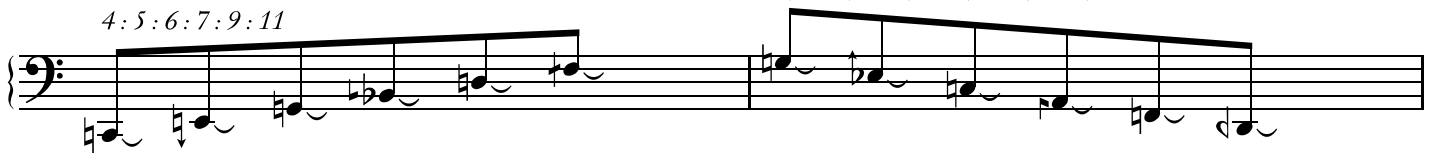


7: 4/3-0

4: 5: 6: 7: 9: 11

8: 1/1-U

1/4: 1/5: 1/6: 1/7: 1/9: 1/11

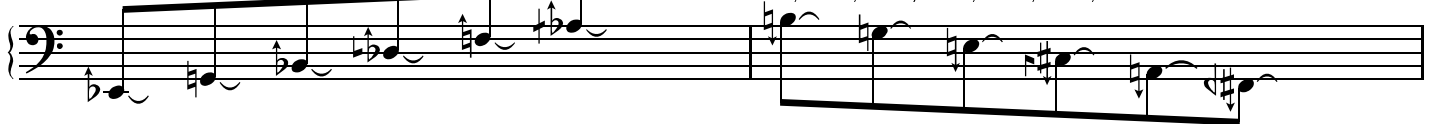


9: 8/5-0

4: 5: 6: 7: 9: 11

10: 5/4-U

1/4: 1/5: 1/6: 1/7: 1/9: 1/11

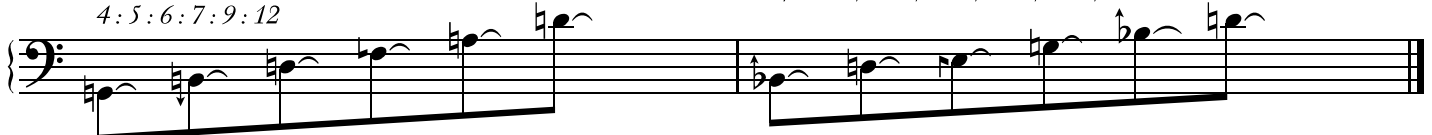


11: 1/1-0

4: 5: 6: 7: 9: 12

12: 3/2-U

1/10: 1/8: 1/7: 1/6: 1/5: 1/4



Appendix C

Dark Brother: HEJI transcription

DARK BROTHER

Final Two Paragraphs from Thomas Wolfe's "God's Lonely Man"

Harry Partch (1942/3)

edited and transcribed by Thomas Nicholson (2019/20)

♩ = 60

Adapted Viola

Voice

Chromelodeon I
(resulting)

Open Z-A^R-X-⁶₅

Kithara I
(first tuning)

Bass Marimba



Ch I

5

7

Ch I

Kth

B M

11

Ch I

Kth

B M

marked rit.....(a tempo)

ff

13 [mute]

A Vla

V

Ch I

Kth

B M

Close Z-X-⁶₅
Open A^L

But the



19

A Vla

V

Ch I

old re - fu - sals drop a - way, the old a - vow - als stand

DARK BROTHER (Partch)

23

A Vla

V

Ch I

and we who were dead have ri - sen we who were lost are found a -

26

A Vla

V

Ch I

- gain, and we who sold the tal - ent, the pas - sion

29

A Vla

V

Ch I

and be - lief of youth in - to the keep - ing of the fless - less dead

31

A Vla

V

Ch I

un - til our hearts were cor - rupt - ed, our tal - ent was - ted

35

A Vla

V

Ch I

and our hope gone, have won our lives back blood - i - ly

39

A Vla

V

Ch I

in sol - i - tude and dark - ness:

DARK BROTHER (Partch)

43

V

and we know that things will be for us as they have been,

Ch I

44

A Vla

V

and we see a - gain as we saw once,

Ch I

Open X-⁶₅

48

A Vla

V

the im - age of the shin - ing cit - y. Far flung,

Ch I

53

A Vla

V

Ch I

and blaz - ing in - to tiers of jew - elled light,—

56

A Vla

V

Ch I

it burns for - ev - er in our vis - ion as we walk the Bridge,—

ff

57

A Vla

V

Ch I

and strong tides are bound round it

Close X-⁶₅

61

A Vla

V

Ch I

and the great ships call. And we walk the Bridge, al - ways we walk the Bridge

66

A Vla

V

Ch I

a - lone with you, — stern friend, the one to whom we speak,

72

A Vla

V

Ch I

rit.....

who ne - ver failed us. Hear: —

76 [Take off mute](a tempo)

A Vla

Ch I

Kth

B M

Close ^RA
Open Z-⁶₅

ten.

78

A Vla

Ch I

Kth

B M

12 12 12 simile

[78]

A Vla

Ch I

Kth

B M

Measures 78-85. A Vla: Melodic line with many accidentals. Ch I: Complex texture with many accidentals. Kth: Rhythmic pattern of chords with downward strokes. B M: Simple bass line of quarter notes.

[78]

A Vla

Ch I

Kth

B M

Measures 86-93. A Vla: Melodic line with many accidentals. Ch I: Complex texture with many accidentals. Kth: Rhythmic pattern of chords with downward strokes. B M: Simple bass line of quarter notes.

79

A Vla

V

Ch I

Kth

B M

Lone - li - ness for - ev - er and the earth

Detailed description: This block contains the musical notation for measures 79 and 80. The staves are arranged vertically. The A Vla staff has a few notes. The V staff has a series of notes. The Ch I staff has a complex, dense texture with many notes. The Kth staff has a few notes. The B M staff has a few notes. The lyrics are written below the V staff.

80

A Vla

V

Ch I

Kth

B M

a - gain Dark bro - - ther and stern

Detailed description: This block contains the musical notation for measures 80 and 81. The staves are arranged vertically. The A Vla staff has a few notes. The V staff has a series of notes. The Ch I staff has a complex, dense texture with many notes. The Kth staff has a few notes. The B M staff has a few notes. The lyrics are written below the V staff.

82

A Vla

V

friend

Ch I

Kth

B M

83

A Vla

V

im - mor - tal face of dark - ness and of

Ch I

Kth

B M

84

A Vla

V

Ch I

Kth

B M

night, with whom the



[85]

A Vla

V

Ch I

Kth

B M

half part of my life— was spent,

87

A Vla

V

Ch I

Kth

B M

and with whom I shall a - bide now till my

Detailed description: This block contains the musical notation for measures 87 and 88. The staves are arranged vertically: A Vla (top), V (vocal), Ch I (harp), Kth (keyboard), and B M (bass). The key signature has one flat (B-flat). The time signature is common time (C). The vocal line (V) has lyrics 'and with whom I shall a - bide now till my'. The harp (Ch I) and keyboard (Kth) parts provide harmonic support. The bass (B M) part has a single note in measure 88.

89

A Vla

V

Ch I

death for - ev - er.

Detailed description: This block contains the musical notation for measure 89. The staves are arranged vertically: A Vla (top), V (vocal), and Ch I (harp). The key signature has one flat (B-flat). The time signature is common time (C). The vocal line (V) has lyrics 'death for - ev - er.'. The harp (Ch I) part provides harmonic support. The A Vla part has a single note in measure 89.

91

A Vla

Ch I

Kth

B M



92

A Vla

V

Ch I

Kth

B M

What___ is there for me to fear as long as you are with me?

94

A Vla

V

Ch I

Kth

B M

Her - o - ic friend,



97

A Vla

V

Ch I

Kth

B M

blood bro - ther of my life dark face

99

A Vla

V

Ch I

Kth

B M

have we not gone to - ge - ther down a mil - lion ways,



101

A Vla

V

Ch I

Kth

B M

have we not coursed to - ge - ther the great and fur - i - ous av - en - ues of night

102

A Vla

V

have we not crossed the storm - y seas a - lone

Ch I

Kth

B M



104

A Vla

V

and known strange lands,

Ch I

Kth

B M

107

A Vla

V

Ch I

Kth

B M

and come a - gain to walk the con - ti - nent of the night



109

A Vla

V

Ch I

Kth

B M

and list - en to the si - lence of the earth?

111

A Vla

V

Ch I

Kth

B M

Have we not been brave and glo - ri - ous when we were to - geth - er, friend?



113

A Vla

V

Ch I

Kth

B M

Have we not known tri - umph, joy and glor - y on this earth,

115

A Vla

V

Ch I

Kth

B M

and will it not be a - gain with me as it was then,



117

A Vla

Ch I

Kth

B M



118

A Vla

V

Ch I

Kth

B M

if you come back to me? Come to me, bro - ther,



121

A Vla

V

Ch I

in the watch - es of the night.

124

A Vla

V

Ch I

Kth

B M

Come to me in the se - cret and most si - lent heart of dark - ness.



126

A Vla

V

Ch I

Kth

B M

Come to me as you al - ways came,

129

A Vla

V

Ch I

Kth

B M

bring - ing to me a - gain the old in - vinc - i - ble strength,



132

A Vla

V

Ch I

Kth

B M

the death - less hope, the tri - um - phant joy and con - fi - dence that will

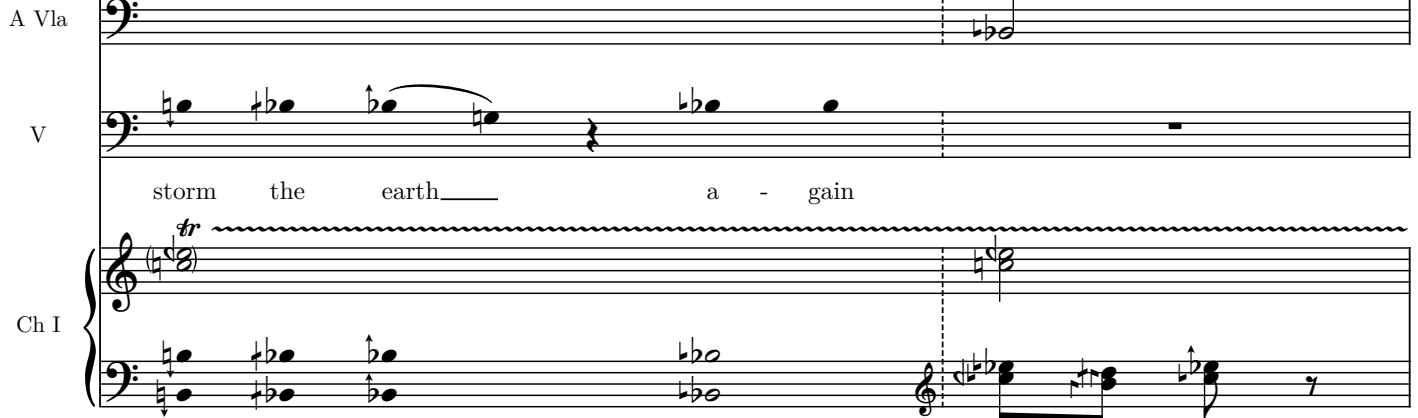
[134]

A Vla

V

Ch I

storm the earth a - gain



136

A Vla

Ch I

Kth

B M



138

A Vla

Kth

B M



Appendix D

Catalogue of epimoric enharmonic proximities

LEGEND

a bold row number: the enharmonic proximity is also in Sabat's *Three Tables for Bob*

a grey box: the enharmonic proximity is one of the most salient

a black box: the enharmonic proximity has potential for salience

No.	Ratio $a : b$	Cents	Powers of primes > 3							HEJI example
			5	7	11	13	17	19	23	
1	4374:4375	0.4	4	1	0	0	0	0	0	$\sharp B$ to $\flat B$
2	384:385	4.5	1	1	1	0	0	0	0	$\flat A$ to $\sharp A$
3	440:441	3.93	1	2	1	0	0	0	0	$\sharp B$ to $\flat C$
4	539:540	3.21	1	2	1	0	0	0	0	$\sharp E$ to $\sharp D$
5	3024:3025	0.57	2	1	2	0	0	0	0	$\flat A$ to $\sharp G$
6	9800:9801	0.18	2	2	2	0	0	0	0	$\sharp F$ to $\sharp E$
7	324:325	5.34	2	0	0	1	0	0	0	$\sharp G$ to $\flat G$
8	624:625	2.77	4	0	0	1	0	0	0	$\flat C$ to $\sharp B$
9	728:729	2.38	0	1	0	1	0	0	0	$\flat G$ to $\sharp F$
10	4095:4096	0.42	1	1	0	1	0	0	0	$\sharp C$ to $\sharp C$
11	350:351	4.94	2	1	0	1	0	0	0	$\sharp G$ to $\flat G$
12	351:352	4.93	0	0	1	1	0	0	0	$\flat G$ to $\sharp G$
13	2079:2080	0.83	1	1	1	1	0	0	0	$\sharp A$ to $\flat A$
14	1000:1001	1.73	3	1	1	1	0	0	0	$\flat G$ to $\flat A$
15	1715:1716	1.01	0	3	1	1	0	0	0	$\flat C$ to $\flat B$
16	363:364	4.76	0	1	2	1	0	0	0	$\sharp G$ to $\flat A$
17	675:676	2.56	2	0	0	2	0	0	0	$\sharp E$ to $\flat F$
18	4224:4225	0.41	2	0	1	2	0	0	0	$\flat D$ to $\flat D$
19	1700:1701	1.02	2	1	0	0	1	0	0	$\flat G$ to $\sharp F$
20	1224:1225	1.41	2	2	0	0	1	0	0	$\sharp E$ to $\flat F$
21	2499:2500	0.69	4	2	0	0	1	0	0	$\sharp F$ to $\flat E$
22	5831:5832	0.3	0	3	0	0	1	0	0	$\flat E$ to $\sharp D$

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Catalogue of epimoric enharmonic proximities – *cont'd from previous page*

No.	Ratio $a : b$	Cents	Powers of primes > 3							HEJI example
			5	7	11	13	17	19	23	
23	374:375	4.62	3	0	1	0	1	0	0	$\sharp\sharp D$ to $\sharp D$
24	594:595	2.91	1	1	1	0	1	0	0	$\sharp E$ to $\sharp E$
25	560:561	3.09	1	1	1	0	1	0	0	$\sharp E$ to $\sharp D$
26	1088:1089	1.59	0	0	2	0	1	0	0	$\sharp A$ to $\sharp A$
27	2057:2058	0.84	0	3	2	0	1	0	0	$\sharp\sharp C$ to $\sharp E$
28	441:442	3.92	0	2	0	1	1	0	0	$\sharp G$ to $\sharp F$
29	832:833	2.08	0	2	0	1	1	0	0	$\sharp B$ to $\sharp B$
30	1274:1275	1.36	2	2	0	1	1	0	0	$\sharp G$ to $\sharp G$
31	935:936	1.85	1	0	1	1	1	0	0	$\sharp F$ to $\sharp G$
32	2430:2431	0.71	1	0	1	1	1	0	0	$\sharp C$ to $\sharp C$
33	714:715	2.42	1	1	1	1	1	0	0	$\sharp G$ to $\sharp G$
34	1155:1156	1.5	1	1	1	0	2	0	0	$\sharp E$ to $\sharp D$
35	2600:2601	0.67	2	0	0	1	2	0	0	$\sharp F$ to $\sharp E$
36	512:513	3.38	0	0	0	0	0	1	0	$\sharp D$ to $\sharp D$
37	1215:1216	1.42	1	0	0	0	0	1	0	$\sharp E$ to $\sharp F$
38	399:400	4.33	2	1	0	0	0	1	0	$\sharp B$ to $\sharp A$
39	342:343	5.05	0	3	0	0	0	1	0	$\sharp G$ to $\sharp A$
40	2375:2376	0.73	3	0	1	0	0	1	0	$\sharp B$ to $\sharp B$
41	1539:1540	1.12	1	1	1	0	0	1	0	$\sharp E$ to $\sharp E$
42	3135:3136	0.55	1	2	1	0	0	1	0	$\sharp A$ to $\sharp B$
43	1728:1729	1	0	1	0	1	0	1	0	$\sharp B$ to $\sharp C$

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Catalogue of epimoric enharmonic proximities – *cont'd from previous page*

No.	Ratio $a : b$	Cents	Powers of primes > 3							HEJI example
			5	7	11	13	17	19	23	
44	455:456	3.8	1	1	0	1	0	1	0	$\sharp\sharp C$ to $\natural C$
45	6174:6175	0.28	2	3	0	1	0	1	0	$\flat\flat B$ to $\sharp\sharp A$
46	494:495	3.5	1	0	1	1	0	1	0	$\flat D$ to $\sharp\sharp C$
47	2925:2926	0.59	2	1	0	2	0	1	0	$\sharp\sharp G$ to $\flat\flat A$
48	5928:5929	0.29	0	2	2	1	0	1	0	$\flat A$ to $\flat\flat A$
49	1520:1521	1.14	1	0	0	2	0	1	0	$\flat E$ to $\sharp\sharp E$
50	475:476	3.64	2	1	0	0	1	1	0	$\sharp G$ to $\flat\sharp G$
51	5984:5985	0.29	1	1	1	0	1	1	0	$\flat\sharp G$ to $\flat\sharp A$
52	968:969	1.79	0	0	2	0	1	1	0	$\flat\sharp C$ to $\flat\sharp C$
53	4199:4200	0.41	2	0	1	1	1	1	0	$\flat\sharp D$ to $\flat\sharp D$
54	2431:2432	0.71	0	0	1	1	1	1	0	$\flat\sharp E$ to $\flat F$
55	360:361	4.8	1	0	0	0	0	2	0	$\sharp G$ to $\flat\flat A$
56	5775:5776	0.3	2	1	1	0	0	2	0	$\flat\sharp G$ to $\flat\flat A$
57	3249:3250	0.53	3	0	0	1	0	2	0	$\flat\flat C$ to $\sharp\sharp B$
58	1444:1445	1.2	1	0	0	0	2	2	0	$\flat\flat A$ to $\flat\sharp\sharp F$
59	575:576	3.01	2	0	0	0	0	0	1	$\flat\sharp F$ to $\flat G$
60	735:736	2.35	1	2	0	0	0	0	1	$\sharp\sharp C$ to $\flat\sharp B$
61	2024:2025	0.86	2	0	1	0	0	0	1	$\flat D$ to $\sharp\sharp D$
62	8624:8625	0.2	3	2	1	0	0	0	1	$\flat\flat E$ to $\flat\sharp\sharp C$
63	483:484	3.58	0	1	2	0	0	0	1	$\flat\sharp C$ to $\flat\sharp C$
64	896:897	1.93	0	1	0	1	0	0	1	$\flat\sharp C$ to $\flat\sharp B$

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Catalogue of epimoric enharmonic proximities – *cont'd from previous page*

No.	Ratio <i>a : b</i>	Cents	Powers of primes > 3							HEJI example
			5	7	11	13	17	19	23	
65	8280:8281	0.21	1	2	0	1	0	0	1	$\sharp\flat C$ to $\flat\sharp E$
66	1287:1288	1.34	0	1	1	1	0	0	1	$\sharp F$ to $\sharp F$
67	3887:3888	0.45	0	0	0	2	0	0	1	$\flat\sharp E$ to $\sharp\flat E$
68	3380:3381	0.51	1	2	0	2	0	0	1	$\flat\flat\sharp B$ to $\sharp B$
69	506:507	3.42	0	0	1	2	0	0	1	$\sharp F$ to $\flat\flat G$
70	459:460	3.77	1	0	0	0	1	0	1	$\sharp C$ to $\sharp C$
71	391:392	4.42	0	2	0	0	1	0	1	$\sharp\sharp G$ to $\flat\flat B$
72	3519:3520	0.49	1	0	1	0	1	0	1	$\sharp\sharp F$ to $\sharp\flat G$
73	390:391	4.43	1	0	0	1	1	0	1	$\flat\sharp A$ to $\sharp\sharp G$
74	1104:1105	1.57	1	0	0	1	1	0	1	$\sharp D$ to $\flat\flat\sharp D$
75	1495:1496	1.16	1	0	1	1	1	0	1	$\sharp\flat\sharp G$ to $\sharp\sharp G$
76	2023:2024	0.86	0	0	2	0	2	0	1	$\sharp\sharp\sharp C$ to $\sharp C$
77	5082:5083	0.34	0	1	2	1	1	0	1	$\sharp\sharp F$ to $\sharp\flat\sharp E$
78	874:875	1.98	3	1	0	0	0	1	1	$\sharp B$ to $\sharp\flat B$
79	1862:1863	0.93	0	2	0	0	0	1	1	$\flat\flat D$ to $\sharp B$
80	759:760	2.28	1	0	1	0	0	1	1	$\sharp\flat E$ to $\flat F$
81	2299:2300	0.75	2	0	2	0	0	1	1	$\sharp\flat\sharp E$ to $\flat F$
82	1196:1197	1.45	0	1	0	1	0	1	1	$\flat\sharp\flat E$ to $\sharp\flat D$
83	2184:2185	0.79	1	1	0	1	0	1	1	$\flat E$ to $\sharp\flat\sharp D$
84	7865:7866	0.22	1	0	2	1	0	1	1	$\flat\sharp\sharp C$ to $\sharp\sharp C$
85	2736:2737	0.63	0	1	0	0	1	1	1	$\sharp G$ to $\sharp\sharp F$

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Catalogue of epimoric enharmonic proximities – *cont'd from previous page*

No.	Ratio <i>a : b</i>	Cents	Powers of primes > 3							HEJI example
			5	7	11	13	17	19	23	
86	3059:3060	0.57	1	1	0	0	1	1	1	$\sharp\flat A$ to $\flat\sharp G$
87	4692:4693	0.37	0	0	0	1	1	2	1	$\sharp\flat D$ to $\flat\sharp F$